Nonminimal Higgs Inflation and Initial Conditions in Cosmology

非最小希格斯暴胀与宇宙学中的初始条件

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To the memory of V.A.Rubakov

谨以此文纪念 V.A. 鲁巴科夫

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Abstract

摘要

We discuss applications of perturbative quantum gravity in the theory of very early quantum Universe and quantum cosmology. Consistency of the theoretical formalism for quantum effects of matter and correspondence with observational status of modern precision cosmology impose stringent bounds on and establish strong links with high-energy particle phenomenology. Within this line of reasoning we study various aspects of one-loop approximation for the cosmological wave function, review Higgs inflation model intertwining the physics of the electroweak sector of the Standard Model with the characteristics of the observable cosmic microwave background and, finally, consider the problem of quantum initial conditions for inflationary Universe. We formulate a cosmological quantum state in the form of the microcanonical density matrix - a universal equipartition of eigenstates of the Wheeler-DeWitt equations. We demonstrate elimination of the inalienable infrared catastrophe of vanishing cosmological constant for the no-boundary quantum state of the Universe and derive initial conditions for inflation in the form of a special garland-type cosmological instanton - the saddle point of quantum gravity path integral. Applied to the cosmological model of the Universe with a hidden sector of numerous conformally invariant higher-spin fields, this setup suggests a solution to the problem of hierarchy between the Planck and the inflation energy scales and, thus, admits applicability of perturbative semiclassical expansion methods.

我们探讨微扰量子引力在极早期量子宇宙理论和量子宇宙学中的应用。物质量子效应理论框架的自洽性，以及与现代精确宇宙学观测现状的一致性，对高能粒子唯象学施加了严格限制，并与之建立了紧密联系。基于这一思路，我们研究了宇宙波函数单圈近似的多个方面，综述了将标准模型电弱区物理与可观测宇宙微波背景特征关联起来的希格斯暴胀模型，最后讨论了暴胀宇宙的量子初始条件问题。我们以微正则密度矩阵——惠勒-德维特方程本征态的通用等概率分布——的形式构造了宇宙学量子态。我们证明了宇宙无边界量子态可以消除宇宙常数为零的固有红外灾变，并以特殊的花环型宇宙瞬子——量子引力路径积分的鞍点——的形式推导出暴胀的初始条件。该框架应用于包含大量共形不变高自旋场隐藏区的宇宙学模型后，解决了普朗克能标与暴胀能标之间的等级问题，因此使得微扰半经典展开方法可以适用。

Keywords

关键词

Quantum cosmology · Higgs inflation · Initial quantum state of the Universe

量子宇宙学 · 希格斯暴胀 · 宇宙初始量子态

Introduction

引言

Quantum cosmology is an exciting field of modern theoretical physics. To begin with, it is an integral part of the unified theory of elementary particles and fundamental interactions. As an application of quantum theory to the Universe as a whole, quantum cosmology is an inalienable part of quantization of gravity. Understanding the structure of physical interactions at the most fundamental microscopic scale of space and time, as is widely agreed now, is the prerogative of the advanced superstring theory motivated by intrinsic mathematical consistency and beauty along with the necessity of circumventing the known difficulties of quantizing Einstein's general relativity. The microscopic realm of this problem seems to exclude quantum cosmology as a science whose field of application is the Universe in all its entirety. However, the quantum origin of the early Universe is what brings macroscopic and microscopic phenomena together tightly intertwining their methods of description and, thus, transcending the commonly accepted physics principle of separation of scales.

量子宇宙学是现代理论物理学中一个令人振奋的研究领域。首先，它是基本粒子与基本相互作用统一理论中不可或缺的组成部分。作为量子理论应用于整个宇宙的成果，量子宇宙学是引力量子化不可分割的一部分。目前学界普遍认为，理解时空最基本微观尺度下物理相互作用的结构是高级超弦理论的研究范畴，超弦理论的发展动机来源于其内在的数学自洽性与美感，同时也源于克服爱因斯坦广义相对论量子化已知难题的现实需求。这一问题的微观领域似乎将量子宇宙学排除在外，因为后者的研究范畴是整个宇宙。然而，早期宇宙的量子起源将宏观现象与微观现象紧密结合，使二者的描述方法深度交织，从而超越了物理学中公认的标度分离原理。

On the other hand, modern cosmology has overstepped the limits of the state of art and started profoundly describing various aspects of the evolution of the very early and present-day Universe. As soon as it entered in the early nineties the stage of precision cosmology, it has become now observational science with a very solid experimental status. In particular, it deeply explained the phenomenology of hot Universe originating from the stage of a very fast quasi-exponential inflationary expansion. However, inflationary cosmology does not describe the initial quantum state of the Universe - the primary goal of quantum cosmology. This is again the point at which complementary fields of physics embrace and call for unification of the presently observable large-scale structure of the Universe with its quantum origin laying the imprint on this structure.

另一方面，现代宇宙学已经突破了原有研究的边界，开始深入描述极早期宇宙与现今宇宙演化的各个方面。自上世纪 90 年代初进入精确宇宙学阶段以来，现代宇宙学如今已经成为拥有扎实实验基础的观测科学。它尤其深入阐释了热宇宙的唯象学，而热宇宙起源于极快的准指数暴胀膨胀阶段。但暴胀宇宙学并不描述宇宙的初始量子态——这正是量子宇宙学的核心目标。这再一次体现了不同物理学分支的互补性，要求我们将目前可观测的宇宙大尺度结构，与其在该结构上留下印记的量子起源统一起来。

One can say that modern quantum cosmology began in 1967 when the famous Wheeler-DeWitt equation was put forward in [1]. This equation arises as a result of application to gravity of the Dirac quantization procedure for systems with canonical constraints [2]. Briefly this equation, or it would be better to say the infinite systems of these equations - four ones per spatial point - looks like

可以说，现代量子宇宙学诞生于 1967 年，那一年文献 [1] 提出了著名的惠勒-德维特方程。该方程是对带正则约束的系统应用狄拉克量子化程序处理引力得到的结果 [2]。我们可以简要给出这个方程——更准确地说，是这无穷多组方程，每个空间点对应 4 个方程——其形式如下

$$/\mathrm{d}\ |\Psi\rangle = 0, \tag{1}$$

where $|\Psi\rangle$ is a quantum state of the Universe, which is sometimes called a wave function of the Universe, while $\hat{\mathcal{H}}$ is the set of Hamiltonian and momentum constraints of the canonical formalism of gravity theory. These constraints depend on spatial components of the spacetime metric and their conjugated momenta along with the full set of degrees of freedom of matter fields. This system is very complicated and its precise mathematically rigorous and explicit formulation is still missing because it involves unsolved issues of operator realization, operator ordering and consistency manageable only at the formal level.

其中 $|\Psi\rangle$ 是宇宙的量子态，有时也被称为宇宙波函数，$\hat{\mathcal{H}}$ 是引力理论正则形式中哈密顿约束与动量约束的集合。这些约束依赖于时空度规的空间分量及其共轭动量，同时也依赖于物质场所有自由度的完整集合。该方程组十分复杂，目前仍不存在数学上严格精确的显式表述，因为它涉及算符实现、算符排序与仅能在形式层面处理的自洽性等未解决问题。

But, as it often happens in physics, the absence of full rigourousness does not prevent from fruitful applications of imperfect mathematical tools by means of their extrapolation from those areas where they were well defined and productively exploited (Sometimes it is even stated that the Wheeler-DeWitt equation is the most useless equation in physics. But this statement is certainly incorrect because, even if this equation is not directly used in concrete quantum gravitational applications, it still fundamentally underlies the results obtained by alternative methods, which are intrinsically equivalent to the Wheeler-DeWitt equation formalism. As a comparison one can point out at the Schroedinger equation which is hardly usable as a tool of high-energy relativistic scattering but it fully underlies scattering phenomenology.). As such a tool we will use the path integration method which gives a formal solution to the Wheeler-DeWitt equations [3-5]. Though this method does not fully resolve such conceptual problems of the formalism as the problem of time, probabilistic interpretation of the cosmological wave function [5, 6], etc., it allows one to reach interesting predictions on the quantum origin of the Universe and its further evolution within a reliable scheme of covariant effective or renormalizable quantum field theory. Within semiclassical expansion, this approach incorporates the concept of quantum-mechanical tunnelling or the 'birth or tunnelling of the Universe from nothing' as a natural interpretation of complex saddle points of the underlying path integral - transition to the physical Lorentzian signature spacetime from the Euclidean manifold describing the classically forbidden state of the gravitational field in imaginary time.

但正如物理学中常见的情况, 缺乏完全的严谨性并不妨碍我们对不完善的数学工具进行富有成果的应用——我们可以将这些工具从定义清晰、应用成熟的领域外推使用 (甚至有时有人称惠勒-德维特方程是物理学中最没用的方程。但这个说法显然不对，因为即使该方程没有直接应用在具体的量子引力问题中, 它依然是替代方法所得结果的基础, 这些方法本质上都等价于惠勒-德维特方程形式体系。举个类比，薛定谔方程作为高能相对论散射的研究工具几乎无法使用，但它完全是散射唯象学的基础)。我们将采用路径积分法作为研究工具，它可以给出惠勒-德维特方程的形式解 [3-5]。尽管该方法没有完全解决形式体系中的概念问题, 比如时间问题、宇宙波函数的概率诠释 [5,6] 等，但它仍能在协变有效或可重整化量子场论的可靠框架内, 得到关于宇宙量子起源及其后续演化的有趣预言。在半经典展开框架下，该方法将量子隧穿即“宇宙无中生有隧穿诞生”的概念纳入体系，将其作为基础路径积分复鞍点的自然诠释——即描述引力场经典禁戒态的欧几里得流形在虚时间下，跃迁到物理洛伦兹号差时空的过程。

The implementation of this concept is the 'no-boundary' or Hartle-Hawking prescription for the wave function of the Universe [3, 4] . It is based on the direct application of the Euclidean quantum field theory to the full system of gravitational and matter fields in the Universe. Semiclassically, their quantum state is described in terms of the imaginary time, which is by means of the Euclidean spacetime, so that the corresponding amplitudes and probabilities are weighted by the exponentiated Euclidean gravitational action, $\exp(-S_E)$. The action is calculated on the gravitational instanton - the saddle point of an underlying path integral over Euclidean 4-geometries. This instanton gives rise to Lorentzian signature spacetime by analytic continuation across minimal hypersurfaces, this continuation being interpreted either as quantum tunnelling or as the creation of the Universe from 'nothing'. An immediate difficulty with this picture is the so-called infrared catastrophe of a small cosmological constant Λ - fundamental or effective (which is induced by matter fields and determining the size of the Universe). The simplest realization of the Hartle-Hawking wave function in minisuperspace approximation of the isotropic and homogeneous spacetime, which describes nucleation of the de Sitter Universe from the Euclidean 4-dimensional hemisphere, has the form

这一概念的实现就是宇宙波函数的“无边界”(即哈特-霍金) 处方 [3, 4] 。它基于将欧几里得量子场论直接应用于宇宙中引力场与物质场的完整系统。半经典层面上，它们的量子态用虚时间描述，虚时间对应欧几里得时空，因此相应的振幅和概率由指数化的欧几里得引力作用量加权，$\exp(-S_E)$ 。该作用量在引力瞬子上计算——引力瞬子是欧几里得 4 几何基础路径积分的鞍点。通过对极小超曲面做解析延拓，该瞬子可以生成洛伦兹号差时空，这种延拓可以被解释为量子隧穿，或是宇宙从“无”中创生。这一图景有一个直接困难，即小宇宙学常数 (无论是基本常数还是由物质场诱导、决定宇宙大小的有效常数) 引发的所谓红外灾难 Λ 。各向同性均匀时空的微超空间近似下，描述德西特宇宙从欧几里得四维半球成核的哈特-霍金波函数最简形式如下

$$\Psi_{\mathrm{HH}} \sim \exp(-S_E) = \exp\left(\frac{3\pi}{2G\Lambda}\right). \tag{2}$$

This diverges for $\Lambda \to 0$ because of indefiniteness of the Euclidean gravitational action in Einstein's general relativity. Such a result looks counterintuitive because it predicts as infinitely more probable the quantum birth of the Universe of infinitely big size.

该表达式在 $\Lambda \to 0$ 处发散，原因是爱因斯坦广义相对论中欧几里得引力作用量不具有定性。这个结果有违直觉，因为它预测无穷大尺寸宇宙的量子诞生概率无限更高。

The alternative tunnelling state of the Universe [7-11] is based on the semiclassical solution of the minisuperspace version of the Wheeler-DeWitt equation - that is, the situation when the full set of Wheeler-DeWitt equations (1) is truncated to a single differential equation for the wave function in the finite-dimensional Friedmann metric sector of the full space of 3-geometries. Setting boundary conditions for this equation appropriate for tunnelling through classically forbidden region leads to the expression weighted by the same exponentiated Euclidean gravity action which is, however, taken with the opposite sign,

宇宙的替代隧飞态 [7-11] 基于微超空间版本惠勒-德维特方程的半经典解——也就是说，该框架将整套惠勒-德维特方程 (1) 约化为单个微分方程，描述整个 3 几何空间中弗里德曼度规有限维扇区的波函数。为该方程设定适用于经典禁戒区域隧穿的边界条件后，得到的表达式同样由指数化的欧几里得引力作用量加权，但加权项的符号相反，

$$\Psi_{\mathrm{T}} \sim \exp\left(S_E\right) = \exp\left(-\frac{3\pi}{2G\Lambda}\right). \tag{3}$$

This result looks less counterintuitive, but its essentially minisuperspace setup, which can hardly be formulated outside of the Friedmann metric context, gives a serious ground to doubt the fundamental nature of such a tunnelling construction.

这个结果的反直觉性更弱，但它本质上是微超空间框架，很难脱离弗里德曼度纳语境构建，因此人们有充分理由质疑这种隧穿结构的基本性。

Apart from the foundational issues underlying the prescriptions (2) and (3), they both suffer from the normalizability problem. If the effective cosmological 'constant' is a composite function of some physical fields, like, for example, the effective potential of the inflaton field $\Lambda = V(\phi)$, both expressions are not suppressed in the high-energy domain $V(\phi) \to \infty$ and not normalizable quantum mechanically, $\int_{-\infty}^{\infty} d\phi |\Psi_{\mathrm{HH,T}}(\phi)|^2 = \infty$ (in fact this is a counterpart to the infrared catastrophe - non-integrable singularity of the wave function at $\Lambda = V(\phi) \to 0$ in the no-boundary case). So the formalism does not have intrinsic cut-off protecting semiclassical low-energy physics from the ultraviolet domain where semiclassical methods fail and nonperturbative methods are not yet available.

除了处方 (2) 和 (3) 本身存在的基础问题，二者还都存在归一性问题。如果有效宇宙“常数”是某些物理场的复合函数，例如暴胀子场的有效势 $\Lambda = V(\phi)$，那么两个表达式在高能区都不会被压制 $V(\phi) \to \infty$，在量子力学上也不可归一化，$\int_{-\infty}^{\infty} d\phi |\Psi_{\mathrm{HH,T}}(\phi)|^2 = \infty$ (实际上这是红外灾难的对应产物——无边界情形下波函数在 $\Lambda = V(\phi) \to 0$ 处存在不可积奇点)。因此该形式体系没有内在截断，可以保护半经典低能物理不受紫外域影响，而半经典方法在紫外域失效，非微扰方法目前也无法应用。

The further progress of quantum cosmology was associated with the application of ideas and methods of the perturbative quantum gravity. Here, the word perturbative has several different meanings. The first one is related to the extension beyond minisuperspace models and the inclusion of perturbations of the gravitational and matter fields on top of the Friedmann background [12, 13]. The second aspect of the perturbative quantum gravity consists in the efforts to overcome the tree-level limit of the semiclassical expansion and to consider one-loop corrections to the wave function of the Universe [14-26]. These works were focused on corrections to the no-boundary wave function of the Universe and, in particular, the relation between the covariant Schwinger-DeWitt formalism of local curvature expansion for quantum effective action [27] and calculations on homogeneous spaces based on spectral summation and zeta-function regularization [28].

量子宇宙学的进一步发展与微扰量子引力思想和方法的应用相关。此处“微扰”有几种不同含义：第一，它关联着超出微超空间模型的扩展，纳入弗里德曼背景上引力场与物质场的扰动 [12, 13]。微扰量子引力的第二个方面，在于努力克服半经典展开的树图级极限，考虑宇宙波函数的单圈修正 [14-26]。这些工作聚焦于对无边界宇宙波函数的修正，尤其关注量子有效作用量的局域曲率展开协变施温格-德维特形式体系 [27]，和基于谱求和与 ζ 函数正则化的齐性空间计算 [28] 二者之间的关系。

In [29] the above two perturbative approaches were successfully combined by including the one-loop contribution of cosmological perturbations. The Hartle-Hawking wave function of the Universe was computed in the one-loop approximation and the rather generic effective action algorithm for the probability of inflation - probability distribution of initial conditions for inflation - was obtained. It was, in particular, shown that high-energy normalizability of the cosmological wave function was determined by the anomalous scaling of the quantum theory on the cosmological gravitational instanton. The one-loop mechanism of generating local peaks of the inflation probability was proposed for the inflaton scalar field with a nonminimal gravitational coupling - the model of inflation originally considered at the classical level in [30-34]. Thus, a direct link between quantum cosmology and particle physics content of the Universe was established.

文献 [29] 通过纳入宇宙学扰动的单圈贡献，成功结合了上述两种微扰方法。在单圈近似下计算了哈特尔-霍金宇宙波函数，得到了适用于暴胀概率 (即暴胀初始条件的概率分布) 的相当通用的有效作用量算法。研究特别表明，宇宙波函数的高能可归一性由宇宙引力瞬子上量子理论的反常标度决定。针对非最小引力耦合的暴胀子标量场 (最初在文献 [30-34] 的经典层面研究的暴胀模型)，本文提出了生成暴胀概率局域峰的单圈机制。由此，量子宇宙学与宇宙的粒子物理内容之间建立了直接关联。

Application of the obtained algorithm to tunnelling wave function was considered in [35]. The corresponding quantum scale of inflation was found by observing a sharp probability peak in the distribution function of chaotic inflationary cosmologies driven by a scalar field with a large negative constant ξ of nonminimal interaction [29, 35, 37, 38]. The attempt to justify this result within path integral representation of the tunnelling wave function was undertaken in [36] with regard to peculiarities of analytic continuation from the Euclidean geometry to the Lorentzian signature spacetime. This continuation was earlier studied in [37] along with the derivation of effective equations of motion driving the inflation stage [38].

文献 [35] 研究了将所得算法应用于隧穿波函数的问题。对于由非最小相互作用常数 ξ 为大负值的标量场驱动的混沌暴胀宇宙，研究人员在其分布函数中观测到一个尖锐的概率峰，由此得到了相应的暴胀量子标度 [29, 35, 37, 38]。文献 [36] 尝试在隧穿波函数的路径积分表示下验证这一结果，同时考虑了从欧几里得几何到洛伦兹号差时空解析延拓的特殊性。该延拓过程与暴胀阶段有效运动方程的推导，此前已在文献 [37, 38] 中研究过。

An important issue in quantum cosmology (and generically in quantum theory) is an explanation of the fact that while our world has quantum origin, we observe it in a macroscopic context as a classical object - quantum spreading does not destroy classical behaviour of large bodies and let it be governed by principles of Laplace determinism. A generally agreed explanation of this fact, which otherwise would contradict simplest estimates of quantum mechanics, is the phenomenon of decoherence [39,40]. Decoherence is the effective classicalization of the quantum world which arises due to the interaction of the observed physical variables with an unobservable cloud of degrees of freedom, which is usually called environment. The natural question which arises when one tries to explain the classicalization of a quantum Universe using the decoherence approach is associated with the definition of such an environment. Indeed, in contrast to the usual description of quantummechanical experiment in a laboratory, there is no external environment, because the object of quantum cosmology is the whole Universe. Thus, we should treat a certain part of degrees of freedom as essential observables, while the rest of them should be considered as an environment with subsequent tracing them out within a formalism of reduced density matrix. It is natural to believe that inhomogeneous degrees of freedom play the role of environment while the macroscopic variables, such as a cosmological scale factor or initial value of the inflaton scalar field, should be treated as observables [41]. Under these assumptions the reduced density matrix in cosmology was calculated in [42, 43] by the method of [29, 35] .

量子宇宙学 (乃至一般量子理论) 中的一个重要问题，是解释如下事实: 我们的世界虽源于量子过程，我们在宏观层面却观测到它是经典客体——量子弥散并未破坏大尺度物体的经典行为，仍让其遵循拉普拉斯决定论。若没有这一解释，结果会与量子力学的最简单估计矛盾，而对该事实得到普遍认可的解释就是退相干现象 [39,40]。退相干是量子世界的有效经典化过程，源于被观测物理变量与不可观测的自由度云 (通常称为环境) 的相互作用。当人们尝试用退相干方法解释量子宇宙的经典化时，自然会出现一个与这类环境定义相关的问题。事实上，与实验室中常规量子力学实验描述不同，量子宇宙学的研究对象是整个宇宙，因此不存在外部环境。因此我们应当将一部分自由度视为核心可观测量，将其余自由度视为环境，随后在约化密度矩阵形式体系中对环境自由度求迹。自然的观点是，非均匀自由度扮演环境的角色，而宇宙学标度因子、暴胀子标量场初始值等宏观变量应当被视为可观测量 [41]。在这些假设下，宇宙学中的约化密度矩阵已由 [29, 35] 方法在 [42, 43] 中计算完成。

Further progress in the theory of the early Universe, at the overlap of particle physics, precision cosmology and quantum gravity, is associated with the so-called Higgs inflation theory. Previously mentioned works on the nonminimally coupled gravity scalar field, which plays the role of inflaton, explicitly or implicitly assumed that in cosmological context this field should belong to the scope of the Grand Unification Theory (GUT). A rather challenging idea to identify this field with the Higgs boson (which was not yet discovered at the moment of the publication of this idea) was put forward in [44]. The motivation for this transition from GUT to the electroweak scale was an interesting hypothesis on nontrivial properties of renormalization group flow of the well-known Standard Model, extrapolating to the Planckian scale and somehow circumventing new physics in the GUT domain. However, a real interest in this model critically grew after the observation [45] that such an identification allows one to establish a connection between the observable cosmological parameters, such as the magnitude and spectral index of cosmic microwave background anisotropy, and the value of the Higgs boson mass - the discovery of this first ever known scalar particle being eagerly expected at that time at LHC. The series of the following works [46-50] using the renormalization group approach then gave more precise predictions for the relation between particle physics and cosmology of this Higgs inflation model. The further development of this model and its various versions can be found in the review paper [51].

早期宇宙理论在粒子物理、精确宇宙学与量子引力交叉领域的进一步进展，与所谓希格斯暴胀理论相关。前文提到的作为暴胀子的非最小耦合引力标量场相关研究，都或明或暗地假设在宇宙学背景下该场应属于大统一理论 (GUT) 的范畴。文献 [44] 提出了一个颇具挑战性的观点: 将该场等同于希格斯玻色子 (该观点提出时希格斯玻色子尚未被发现)。这种从大统一理论尺度转向电弱尺度的动机，来自一个有趣的假设: 推广至普朗克尺度的著名标准模型，其重整化群流具有非平凡性质，可以某种方式绕开大统一领域的新物理。然而，在研究 [45] 发现这种等同性可以建立可观测宇宙学参数 (如宇宙微波背景各向异性的幅度和谱指数) 与希格斯玻色子质量之间的关联后——当时人们正迫切期待大型强子对撞机发现这种人类已知的首个标量粒子——学界对该模型的关注度大幅提升。随后一系列采用重整化群方法的研究 [46-50] 给出了该希格斯暴胀模型中粒子物理与宇宙学关联的更精确预言。该模型及其各类变体的进一步发展可参见综述文章 [51]。

The success of the Higgs inflation model, even though it was somewhat marred by controversy of the strong coupling scale [52] (later corrected in [53]), rather persuasively connects cosmology with the Standard Model of particle physics. However, the Standard Model is often understood as an effective field theory, which at Planckian scale should follow from a more fundamental paradigm of the superstring theory. It is well known that the variety of options open by string models - landscape of string vacua - is huge [54, 55] . Thus, it sounds natural to find certain principles restricting this landscape, and these principles might stem not only from the internal logic of the string theory but, in particular, might follow from quantum cosmology.

希格斯暴涨模型尽管一定程度上受强耦合标度争议的影响 [52](该问题后在 [53] 中得到修正)，但其成功颇具说服力地将宇宙学与粒子物理标准模型联系在了一起。然而，标准模型通常被认为是一种有效场论，在普朗克尺度下它应当源自超弦理论这一更基础的范式。众所周知，弦模型开放的诸多选项——弦真空景观——数量极为庞大 [54, 55] 。因此，寻找限制该景观的特定原理是十分自然的，这些原理不仅可能来自弦理论的内在逻辑，尤其还可能源自量子宇宙学。

On the other hand, Higgs inflation does not explain the origin of initial conditions for the inflationary scenario. Obviously, this should be the domain of energy scales where semiclassical physics of quantum gravity and the quantum theory of the standard particle physics model both meet with the frontier of essentially nonperturbative quantum gravity. So, in the absence of nonperturbative methods, the question arises whether we still can rationally describe the quantum origin of the Universe and whether the no-boundary or tunnelling prescriptions for its quantum state can survive extension to relevant energy scales. The answer, of course, depends on the energy scale of this phenomenon - if it is sufficiently below the Planck scale (and the interpretation of the current cosmological data from the Planck satellite [56] is such that it is indeed four, five or even more orders of magnitude below Planckian 10^{19}GeV) than we can hope on the success of our theoretical description. But the energy scale of the quantum birth of the Universe should not be input by hands - to be a reliable element of the theoretical scheme it should follow from the physical model of our Universe and its particle physics contents.

另一方面，希格斯暴涨无法解释暴涨场景初始条件的起源。显然，这属于半经典量子引力物理、标准粒子物理模型量子论与本质非微扰量子引力前沿交汇的能标范畴。因此，在缺乏非微扰方法的情况下，问题就来了: 我们能否依然合理地描述宇宙的量子起源，宇宙量子态的无边界假设或隧穿假设在延伸到相关能标后是否依然成立。答案当然取决于该现象的能标——如果它远低于普朗克尺度 (普朗克卫星当前宇宙学数据的解读 [56] 表明，它确实比普朗克尺度低四、五甚至更多个数量级 10^{19}GeV)，那么我们就可以期待理论描述取得成功。但宇宙量子诞生的能标不应当人为输入；要成为理论框架中可靠的一环，它应当源自我们宇宙的物理模型及其粒子物理内容。

One step in this direction, which might resolve the above issue and simultaneously supply the string theory with possible landscape selection criteria, was undertaken in [57-59]. The main guiding rule in the implementation of this step was due to the Occam razor principle to avoid unnecessary and redundant assumptions on the choice of distinguished quantum states like the no-boundary or tunnelling ones and replace this choice by a universal equipartition - the microcanonical density matrix of the Universe [59]. Initially interpreted in [57, 58] as an extension of the old idea of the quasi-thermal Euclidean quantum gravity density matrix [60], this suggestion was understood in [59] as an equipartition in the space of all wave functions satisfying the system of Wheeler-DeWitt equations. This is the analogue of a conventional microcanonical density matrix for which, however, even the notion of physical time fundamentally arises as an operator-ordering parameter. Application of this idea to the cosmological model with many species of conformally invariant quantum fields leads to a number of remarkable conclusions. In particular, it eliminates the inalienable infrared catastrophe of the no-boundary state, restricts from above the cosmology energy scale - the main ground for a possible landscape selection rule - determines the CMB spectrum and energy scale of inflation [63,64] depending on the tower of conformal particles in the model and, thus, establishes the hierarchy between this scale and the effective Planckian cut-off below which we can safely apply the semiclassical perturbation theory [65]. Other interesting properties relating this model to other gravitational models can be found in [61, 62] .

文献 [57-59] 朝着解决上述问题、同时为弦理论提供可能的景观选择准则迈出了一步。这一步骤的核心指导原则来自奥卡姆剃刀原理: 避免在选择无边界态、隧穿态这类特殊量子态时引入不必要的冗余假设，将这种选择替换为通用等概率分布——宇宙的微正则密度矩阵 [59]。该提议最初在 [57, 58] 中被解读为准热欧几里得量子引力密度矩阵旧思想的延伸 [60]，后在 [59] 中被重新理解为满足惠勒-德维特方程组的所有波函数在空间中的等概率分布。这是常规微正则密度矩阵的类比，但即使是物理时间的概念，也从根本上起源于此作为算符排序参数。将该思想应用于包含多种共形不变量子场的宇宙学模型，可以得到许多值得关注的结论。尤其，它消除了无边界态固有的红外灾难，从上方限制了宇宙学能标 (这是可能的景观选择规则的核心基础)，还能根据模型中共形粒子的能谱确定 CMB 谱和暴涨的能标 [63,64]，从而建立了该能标与有效普朗克截断之间的层级——在该截断以下我们可以安全地使用半经典微扰论 [65]。该模型与其他引力模型相关的其他有趣性质可参见 [61, 62] 。

In this chapter we would like to give a comprehensive review of the most interesting ideas and the methods briefly reviewed above. The structure of this chapter is the following: the second section presents the wave function of the Universe in the one-loop approximation; the third section gives details of the nonminimal Higgs inflation model; in the fourth section the theory of the cosmological density matrix is presented; the last section contains concluding remarks.

在本章中，我们希望对上述最值得关注的思想与方法进行全面综述。本章结构如下: 第二节给出宇宙波函数的单圈近似；第三节详细介绍非最小希格斯暴涨模型；第四节阐述宇宙学密度矩阵理论；最后一节给出结论。

Wave Function of the Universe in the One-Loop Approximation

单圈近似下的宇宙波函数

The general theory of the wave function in quantum cosmology, as a solution of the system of Wheeler-DeWitt equations, and the way how this theory can be embedded in the canonical quantization formalism of constrained dynamical systems was essentially advanced in the series of papers [5, 66-70], including detailed one-loop approximation for quantum Dirac constraints [66], operator-ordering problem and the problem of physical inner product [67, 68] , realization of the path integral method for such systems [69, 70], unitarity in quantum cosmology [5], etc. These works, though they illuminate many hard issues in quantum cosmology, still remain at the formal level of the state of art and do not lead to concrete physical results. One of the reasons is that these works disregard the infinite dimensional nature of gravitational configuration space and rely on manifestly noncovariant canonical formalism incapable of handling the issue of ultraviolet renormalization, quantum anomalies and so on. So, in this section instead of considering formal issues of quantum cosmology, we focus on the one-loop approximation for the cosmological wave function, in which these issues of quantum state normalization, anomalies, etc. arise in full height.

作为惠勒-德维特方程组的解，量子宇宙学中波函数的通用理论，以及该理论如何嵌入约束动力系统的正则量子化形式体系，已在文献 [5, 66-70] 中得到重要推进，这些工作包括: 量子狄拉克约束的详细单圈近似 [66]、算符排序问题与物理内积问题 [67, 68] 、该类系统路径积分方法的实现 [69, 70]、量子宇宙学中的幺正性 [5] 等。尽管这些工作阐明了量子宇宙学中的诸多难题，但仍处于现有研究的形式层面，并未得出具体物理结果。原因之一在于，这些工作忽略了引力构形空间的无穷维性质，依赖显然是非协变的正则形式体系，无法处理紫外重整化、量子反常等问题。因此，本节我们不讨论量子宇宙学的形式问题，而是聚焦于宇宙波函数的单圈近似，量子态归一化、反常等上述问题都会在该近似中完整呈现。

Normalizability of Cosmological Wave Functions

宇宙波函数的可归一性

The no-boundary and tunnelling cosmological wave functions with the contribution of linearized field fluctuations on top of the Friedmann homogeneous metric background read as infinite products of Gaussian (quasi-vacuum) states of harmonic oscillators with a time-dependent frequency parameter,

弗里德曼均匀度规背景下叠加线性化场涨落贡献的无边界宇宙波函数与隧穿宇宙波函数，可表示为含时频率参数谐振子高斯 (准真空) 态的无穷乘积，

$$\Psi(t\mid\varphi,f)=\frac{1}{\sqrt{v_\varphi^*(t)}}\exp\left(\mp I(\varphi)/2+iS(t,\varphi)\right)\times\prod_n\psi_n(t,\varphi\mid f_n),\tag{4}$$

$$\psi_n(t,\varphi\mid f_n)=\frac{1}{\sqrt{v_n^*(t)}}\exp\left(-\frac{1}{2}\Omega_n(t)f_n^2\right),\ \Omega_n(t)=-a^k(t)\frac{\dot v_n^*(t)}{v_n^*(t)}.\tag{5}$$

Here, the sign minus or plus in front of the Euclidean action $I(\varphi)$ in the exponential of (4) corresponds to the no-boundary and to the tunnelling wave functions of the Universe, respectively, and f_n describe amplitudes of inhomogeneous modes, while v_n correspond to positive-frequency solutions of linearized second-order differential equations for these modes. The power k of the cosmological scale factor $a(t)$ in the expression for the function Ω_n depends on the spin s of the field under consideration and on its parametrization. For the ’standard’ parametrization $k=\ 3-2s$. Inclusion of inhomogeneous modes into the wave function of the Universe was first considered in [12,13]. The principal (and rather nontrivial) achievement of [29] was that the diagonal of the reduced density matrix corresponding to the wave function (4)

此处，式 (4) 指数中欧几里得作用量 $I(\varphi)$ 前的负号与正号分别对应宇宙的无边界波函数与隧穿波函数，f_n 描述非均匀模的振幅，v_n 对应这些模的线性化二阶微分方程的正频率解。函数 Ω_n 表达式中宇宙标度因子 $a(t)$ 的幂次 k 取决于所研究场的自旋 s 及其参数化方式。对于“标准”参数化有 $k=\ 3-2s$。宇宙波函数包含非均匀模的研究最早见于文献 [12,13]。文献 [29] 最重要 (且相当不平凡) 的成果是，证明了对应波函数 (4) 的约化密度矩阵对角元

$$\rho(t\mid\varphi)\equiv\rho(t\mid\varphi,\varphi)=\int\prod_n df_n|\Psi(t\mid\varphi,f)|^2\tag{6}$$

can be represented by the one-loop effective action of the full set of inhomogeneous field modes $\Gamma_{1-\text{loop}}$ which admits covariant regularization and renormalization of its UV divergences,

可以由全部非均匀场模 $\Gamma_{1-\text{loop}}$ 的单圈有效作用量表示，该作用量允许对其紫外发散进行协变正则化与重整化，

$$\rho(t\mid\varphi)=\frac{\sqrt{\Delta_\varphi}}{|v_\varphi(t)|}\exp\left(\mp I(\varphi)-\Gamma_{1-\text{loop}}(\varphi)\right).\tag{7}$$

Here

此处

$$\Delta_\varphi\equiv ia^k\left(v_\varphi^*\dot v_\varphi-\dot v_\varphi^*v_\varphi\right)\tag{8}$$

is the Wronskian of the basis functions of the wave operator of the field φ and $\Gamma_{1-\text{loop}}(\varphi)$ is this one-loop effective action calculated on the DeSitter instanton of the radius $1/H(\varphi)$, where $H(\varphi)$ is the effective Hubble parameter as a function of the inflaton field. When $H(\varphi)\to\infty$, the covariantly regularized and renormalized effective action of the infinite set of inhomogeneous quantum modes $\Gamma_{1-\text{loop}}(\varphi)$ has a scaling behaviour

是场 φ 波动算子基函数的朗斯基行列式，$\Gamma_{1-\text{loop}}(\varphi)$ 是半径为 $1/H(\varphi)$ 的德西特瞬子上计算得到的该单圈有效作用量，其中 $H(\varphi)$ 是作为暴胀子场函数的有效哈勃参数。当 $H(\varphi)\to\infty$ 时，无穷多非均匀量子模 $\Gamma_{1-\text{loop}}(\varphi)$ 经协变正则化与重整化后的有效作用量满足标度行为

$$\Gamma_{1-\text{loop}}(\varphi) = Z \ln \frac{H(\varphi)}{\mu}, \tag{9}$$

where Z is the anomalous scaling of the theory, and μ is a renormalization scale. Then the condition of normalizability of the wave function of the Universe in the UV domain reduces to the requirement that the parameter Z should be bounded from below by a positive number depending on the sector of the homogeneous field mode φ . In this particular case this bound is

其中 Z 是该理论的反常标度，μ 是重整化标度。那么宇宙波函数在紫外区域的可归一性条件就等价于要求参数 Z 存在下界，该下界为一个依赖于均匀场模 φ 扇区的正数。在该特殊情形下，这个界为

$$Z > 1 \tag{10}$$

and this condition provides us with the selection criterion for particle physics models [29]. The work [29] was limited to the case of the Hartle-Hawking (no-boundary) wave function, and it was mentioned there that the extension to the model of chaotic inflation driven by a scalar field nonminimally coupled to gravity can bring serious advantages in comparison with the inflation driven by the minimally coupled scalar field.

该条件即为粒子物理模型的选择判据 [29]。文献 [29] 的研究局限于哈特-霍金 (无边界) 波函数情形，文中提到，推广到与引力非最小耦合标量场驱动的混沌膨胀模型，相比最小耦合标量场驱动的暴胀，能带来显著优势。

Comparison of one-loop tunnelling wave function of the Universe with the no-boundary wave function for a nonminimally coupled scalar field [35] begins with the classical Lagrangian in the Jordan frame of fields

非最小耦合标量场情形下，单圈宇宙隧穿波函数与无边界波函数的对比 [35] 从约旦场框架下的经典拉格朗日量出发

$$L(g_{\mu\nu}, \varphi) = g^{1/2}\left[\frac{m_P^2}{16\pi}R - \frac{1}{2}\xi R\varphi^2 - \frac{1}{2}(\nabla\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4\right]. \tag{11}$$

For a negative nonminimal coupling constant $\xi = -|\xi|$, this model can generate a chaotic inflationary scenario with the inflaton potential in the Einstein frame

当非最小耦合常数 $\xi = -|\xi|$ 为负时，该模型可生成具有爱因斯坦框架下暴胀势的混沌暴胀场景

$$U(\phi)|_{\phi=\phi(\varphi)} = \frac{m^2\varphi^2/2 + \lambda\varphi^4/4}{\left(1 + 8\pi|\xi|\varphi^2/m_P^2\right)^2}, \tag{12}$$

and the one-loop approximated probability distributions in the no-boundary and tunnelling states acquire the following form:

无边界态与隧穿态的单圈近似概率分布具有如下形式:

$$\rho_{NB,T}(\varphi) = \exp\left[\pm\frac{3m_P^4}{8U(\phi(\varphi))}\right]\varphi^{-Z-2}. \tag{13}$$

At least naively, this makes both the no-boundary and tunnelling wave functions normalizable at over-Planckian scales provided the parameter Z satisfies the inequality $Z > -1$ serving as a selection criterion for consistent particle physics models with a justifiable semiclassical loop expansion.

只要参数 Z 满足不等式 $Z > -1$ ，至少直观上这就能让无边界波函数和隧穿波函数在超普朗克尺度下均可归一化，$Z > -1$ 是半经典圈展开自洽的合格粒子物理模型的选择判据。

Although the expression (13) is strictly valid only in the limit $\varphi \to \infty$, it can be used for a qualitatively good description at intermediate energy scales. In this domain the distribution (13) can generate the inflation probability peak at $\varphi = \varphi_I$ with the dispersion σ ,

尽管表达式 (13) 仅在 $\varphi \to \infty$ 极限下严格成立，它仍可用于在中等能量尺度给出良好的定性描述。在该能区，分布 (13) 可以在 $\varphi = \varphi_I$ 处产生暴胀概率峰，弥散为 σ ,

$$\varphi_I^2 = \frac{2\,|I_1|}{Z+2},\ \sigma^2 = \frac{|I_1|}{(Z+2)^2},$$

$$I_1 = -24\pi\frac{|\xi|}{\lambda}\,(1+\delta)\,m_P^2,\ \delta = -8\pi\frac{|\xi|\,m^2}{m_P^2}. \tag{14}$$

Here, I_1 is a second coefficient of expansion of the Euclidean action in inverse powers of φ :

此处，I_1 是欧几里得作用量按 φ 负幂次展开的二阶系数:

$$I\,(\varphi) = -\frac{3m_P^4}{U\,(\phi\,(\varphi))} = I_0 + \frac{I_1}{\varphi^2} + O\big(1/\varphi^4\big). \tag{15}$$

For the no-boundary and tunnelling states, this peak exists in complimentary ranges of the parameter δ . For the no-boundary state it can be realized only for $\delta < -1\,(I_1 > 0)$ and, thus, corresponds to the eternal inflation with the field φ on the negative slope of the inflaton potential (12) growing from its starting value φ_I . For a tunnelling proposal this peak takes place for $\delta > -1$ and generates the finite duration of the inflationary stage.

对于无边界态和隧穿态，该峰值存在于参数 δ 的互补范围中。无边界态仅在 $\delta < -1\,(I_1 > 0)$ 条件下可实现，因此对应暴胀子势 (12) 负斜率上场 φ 从初值 φ_I 开始增长的永恒暴胀。对于隧穿方案，该峰值出现在 $\delta > -1$ 条件下，暴胀阶段具有有限的持续时长。

We can ask ourselves how the anomalous scaling Z behaves in the theories with strong negative nonminimal coupling and at large values of the scalar field φ . It was well known [71, 72] that the expression for Z includes the quartic contributions in terms of effective masses of particles present in the model. For a generic theory on a spherical de Sitter background,

我们可以思考，在强负非最小耦合理论和大标量场 φ 取值下，反常标度 Z 会表现出何种性质。众所周知 [71, 72] ，Z 的表达式包含模型中所有粒子有效质量的四次贡献。对于球对称德西特背景下的一般理论，

$$Z = \frac{1}{12H^4}\left(\sum_{\chi} m_\chi^4 + 4\sum_{A} m_A^4 - 4\sum_{\psi} m_\psi^4\right), \tag{16}$$

where H is the inverse de Sitter radius (or the Hubble parameter) and the summation goes over all scalars χ , vector gauge bosons A and Dirac spinors ψ . Their effective masses for large φ are dominated by the contributions $m_\chi^2 = \lambda_\chi \phi^2/2, m_A^2 = g_A \varphi^2$ and $m_\psi^2 = f_\psi^2 \varphi^2$ induced via the Higgs mechanism from their interaction Lagrangian with the inflaton field.

其中 H 是德西特逆半径 (即哈勃参数)，求和覆盖所有标量 χ 、矢量规范玻色子 A 和狄拉克旋量 ψ 。对于大 φ ，其有效质量由希格斯机制从它们与暴胀子场的相互作用拉格朗日量诱导出的贡献 $m_\chi^2 = \lambda_\chi \phi^2/2, m_A^2 = g_A \varphi^2$ 和 $m_\psi^2 = f_\psi^2 \varphi^2$ 主导。

Thus, in view of the relation $\varphi^2/H^2 = 12\,|\xi|\,/\lambda$, we get the leading contribution of large $|\xi|$ to the total anomalous scaling of the theory:

因此，根据关系 $\varphi^2/H^2 = 12\,|\xi|\,/\lambda$ ，我们得到大 $|\xi|$ 对理论总反常标度的领头贡献:

$$Z = 6\frac{\xi^2}{\lambda}\mathbf{A} + O\left(|\xi|\right), \tag{17}$$

where

其中

$$\mathbf{A} = \frac{1}{2\lambda}\left(\sum_{\chi} \lambda_\chi^2 + 16\sum_{A} g_A^4 - 16\sum_{\psi} g_\psi^4\right), \tag{18}$$

which contains the same large dimensionless ratio $\frac{\xi^2}{\lambda}$ and the universal quantity $\mathbf{A}$ determined by a particle physics model.

它包含相同的大无量纲比 $\frac{\xi^2}{\lambda}$ ，以及由粒子物理模型确定的普适量 $\mathbf{A}$ 。

Thus, the consideration of the wave function of the Universe in the one-loop approximation establishes an interesting link between the cosmology and particle physics, it permits to make both the no-boundary and tunnelling wave functions of the Universe normalizable, and for the case of the tunnelling wave function it produces the probability distribution peak predicting initial conditions for inflation.

因此，对宇宙波函数的单圈近似考虑，在宇宙学和粒子物理学之间建立了有趣的关联: 它可以让无边界和隧穿宇宙波函数都满足可归一化，且对于隧穿波函数，它给出了预测暴胀初始条件的概率分布峰。

Decoherence in Quantum Cosmology

量子宇宙学中的退相干

The perturbative technique sketched above also allows one to study the off-diagonal elements of the reduced density matrix of the universe obtained from the no-boundary or tunnelling wave functions [42, 43] by integrating out the inhomogeneous fields. This is interesting because it gives the information about the decoherence of the Universe which is responsible for its classicalization. As we have already mentioned in the Introduction, the role of the environment in the decoherence approach to cosmology is played by the part of the degrees of freedom, which are, generally, higher-order harmonics representing the inhomogeneous cosmological perturbations. Tracing them out we obtain the reduced density matrix. Information about the decoherence behaviour of the system is contained in the off-diagonal elements of its density matrix. In our case they read

上文概述的微扰技术也可用于研究宇宙约化密度矩阵的非对角元，该矩阵是从无边界波函数或隧穿波函数 [42,43] 中积去非均匀场得到的。这项研究很有意义，因为它给出了宇宙退相干的相关信息，而退相干是宇宙经典化的原因。正如我们在引言中已经提到的，在宇宙学退相干方法中，环境的角色由部分自由度承担，这些自由度通常是代表非均匀宇宙学扰动的高阶谐波。对这些自由度求迹后我们得到约化密度矩阵。系统退相干行为的信息包含在其密度矩阵的非对角元中。在我们的研究中，这些非对角元可写为

$$\rho\left(t \mid \varphi, \varphi'\right) = \left(\frac{\Delta_{\varphi}\Delta_{\varphi'}}{v_{\varphi}v^{*}_{\varphi'}}\right)^{1/4} \exp\left(-\frac{1}{2}\Gamma - \frac{1}{2}\Gamma' + i\left(S - S'\right)\right) D\left(t \mid \varphi, \varphi'\right). \tag{19}$$

Here $\Gamma = \Gamma(\varphi)$, $\Gamma' = \Gamma(\varphi')$, S and S' are Lorentzian counterparts to these Euclidean actions corresponding to the overbarrier evolution from the Euclidean-Lorentzian transition points to the points φ and φ' and $D\left(t \mid \varphi, \varphi'\right)$ is the so-called decoherence factor defined by the formula

此处 $\Gamma = \Gamma(\varphi)$, $\Gamma' = \Gamma(\varphi')$, S 和 S' 是对应这些欧几里得作用量的洛伦兹对应量, 描述从欧几里得-洛伦兹转变点到点 φ 和 φ' 的势垒上演化，$D\left(t \mid \varphi, \varphi'\right)$ 是由下式定义的所谓退相干因子

$$D\left(t \mid \varphi, \varphi'\right) = \prod_{n} \left(\frac{4\,\mathrm{Re}\,\Omega_n\,\mathrm{Re}\,\Omega'^{*}_n}{\left(\Omega_n + \Omega'^{*}_n\right)^2}\right)^{\frac{1}{4}} \left(\frac{v_n v'^{*}_n}{v^{*}_n v'_n}\right)^{\frac{1}{4}}. \tag{20}$$

How do you cope with ultraviolet divergences appearing in the infinite product of this type? This question was already discussed in [73-75]. In [42,43] we used the dimensional regularization [76]. As usual, the main effect of a dimensional regularization consists in changing the number of degrees of freedom involved in summation. For example, for a scalar field, the degeneracy number of harmonics in the spacetime of dimensionality d changes from the well-known value (see, e.g., [77]) $\dim(n, 4) = n^2$ to

如何处理这类无穷乘积中出现的紫外发散？该问题已在文献 [73-75] 中讨论过。我们在 [42,43] 中使用了维数正规化 [76]。和通常情况一样, 维数正规化的主要作用是改变参与求和的自由度数量。例如, 对于标量场，维度为 d 的时空中谐波的简并度会从大家熟知的数值 (例如参见 [77]) $\dim(n, 4) = n^2$ 变为

$$\dim(n, d) = \frac{(2n + d - 4)\,\Gamma(n + d - 3)}{\Gamma(n)\,\Gamma(d - 1)}. \tag{21}$$

Making analytical continuation and discarding the poles $1/(d - 4)$, one has finite values for $D\left(t \mid \varphi, \varphi'\right)$. However, for scalar, photon and graviton fields, one gets because of oversubtraction of UV infinities a pathological behaviour:

做解析延拓并舍去极点 $1/(d-4)$ 后，我们得到 $D(t \mid \varphi, \varphi')$ 的有限值。然而，对标量场、光子场和引力子场，对紫外无穷的过度减除会导致病态行为:

$$|D(t \mid \varphi, \varphi')| \to \infty \,, \text{at } |\varphi - \varphi'| \to \infty \,. \tag{22}$$

For example, for a massive scalar field $|D(t \mid \varphi, \varphi')| \approx \exp\left(\frac{7}{64} m^3 \bar{a}(a-a')^2\right)$, where

例如，对有质量标量场有 $|D(t \mid \varphi, \varphi')| \approx \exp\left(\frac{7}{64} m^3 \bar{a}(a-a')^2\right)$，其中

$$a = \frac{1}{H(\varphi)} \cosh H(\varphi)\, t,\ a' = \frac{1}{H(\varphi')} \cosh H(\varphi')\, t,\ \bar{a} = \frac{a+a'}{2}. \tag{23}$$

Such a form of a decoherence factor not only does not correspond to decoherence, but also renders the density matrix ill defined, breaking the condition $\mathrm{Tr}\left(\rho^2\right) \le 1$. However, there is a remedy - using the reparametrization of a bosonic scalar field

这种形式的退相干因子不仅不符合退相干的物理图象,还会使密度矩阵定义不良,破坏条件 $\mathrm{Tr}\left(\rho^2\right) \le 1$ 。不过存在解决方法——使用玻色标量场的重参数化

$$f \to \widetilde{f} = a^{\mu} f,\ v_n \to \tilde{v}_n = a^{\mu} v_n \tag{24}$$

one can get the new set of frequency functions

可以得到一组新的频率函数

$$\Omega_n(t) = -i a^{3-2\mu}(t) \frac{\dot{\tilde{v}}_n^*(t)}{\tilde{v}_n^*(t)}. \tag{25}$$

In such a way one can suppress ultraviolet divergences. For the so-called conformal parametrization, $\mu = 1$, for the massive scalar field one has

通过这种方法可以压制紫外发散。对于所谓共形参数化，即 $\mu = 1$ ，对有质量标量场可以得到

$$\left|\widetilde{D}(t \mid \varphi, \varphi')\right| = \exp\left(-\frac{m^3 \pi \bar{a}(a-a')^2}{64}\right), \tag{26}$$

For the case of fermions this method does not work [43]. The wave function of the Universe filled with fermions has the form [43, 78, 79]

对于费米子，该方法不生效 [43]。充满费米子的宇宙波函数形式为 [43, 78, 79]

$$\Psi(t, \varphi \mid x, y) = \Psi_0(t, \varphi) \prod_n \psi_n(t \mid x_n, y_n), \tag{27}$$

$$\psi_n(t \mid x_n, y_n) = v_n - \frac{i\dot{v}_n + \nu v_n}{m} x_n y_n, \tag{28}$$

where x and y are Grassmann variables and partial wave functions read in terms of relevant basis functions v_n satisfying the second-order equation:

其中 x 和 y 是格拉斯曼变量，分波函数可以用满足二阶方程的相关基函数 v_n 表示为:

$$\ddot{v}_n + \left(-i\dot{v} + m^2 + v^2\right) v_n = 0,\ v = \frac{n+\frac{1}{2}}{a}. \tag{29}$$

The corresponding result reads in terms of the exponentiated divergent sum [79]

相应的结果可以用指数化的发散和表示 [79]

$$|D\left(a, \varphi \mid a', \varphi'\right)| = \exp\left(-\frac{m^2 (a-a')^2}{8} \sum_{n=1} \frac{n\,(n+1)}{\left(n+\frac{1}{2}\right)^2}\right), \tag{30}$$

whose dimensional regularization can be done using the fact that for spinors in d - dimensional spacetime the degeneracy of the spectrum eigenvalues is equal to

它的维数正规化可以利用以下事实完成: 对于 d 维时空的旋量，谱本征值的简并度等于

$$\dim\left(n, d\right) = \frac{\Gamma\left(n + 2^{(d-2)}\right) \Gamma\left(n + 2^{(d-2)/2} - 1\right)}{\left[\Gamma\left(2^{(d-2)/2}\right)\right]^2 \Gamma\left(n+1\right) \Gamma\left(n\right)}. \tag{31}$$

Still the renormalization procedure fails, like in the boson case, because it gives a non-integrable kernel of the reduced density matrix with growing off-diagonal elements,

和玻色子情况一样，重整化过程仍然失败，因为它得到的约化密度矩阵核不可积，且非对角元不断增长,

$$|D\left(a, \varphi \mid a', \varphi'\right)| = \exp\left(-m^2 (a-a')^2 I\right), \tag{32}$$

having a negative finite constant $I < 0$. Moreover, one cannot use the conformal reparametrization in this case because standard fermion variables are already presented in the conformal parametrization.

并且存在负的有限常数 $I < 0$ 。此外，这种情况无法使用共形重参数化，因为标准费米子变量已经是共形参数化的形式了。

However, there is another way to circumvent this problem. One can perform a non-local Bogoliubov transformation mixing Grassmann variables x and y . This transformation modifies the effective mass of fermions in equation (29) for their basis functions. Choosing it in a certain way one can suppress ultraviolet divergences. The reasonable idea is to fix this transformation by the requirement that the decoherence is absent in a static spacetime. Then one gets a satisfactory result (32) with a positive finite I .

不过，还有另一种方法可以规避该问题: 可以对混合的格拉斯曼变量 x 和 y 执行非局部博戈留波夫变换。该变换会修改方程 (29) 中费米子基函数的有效质量。通过特定方式选取该变换即可压制紫外发散。一个合理的思路是，以静态时空中不存在退相干为要求固定该变换，随后我们就能得到令人满意的结果 (32)，对应一个正的有限值 I 。

All these examples imply strong dependence on parameterization of quantum variables and cast certain doubt on the physical significance of the results. However, these examples show that consistency of the reduced density matrix might determine the very definition of the environment in a quantum cosmological model and serve as a criterion of the choice of observable (pointer) basis in field-theoretical systems - a serious issue of the decoherence theory.

所有这些例子都表明结果对量子变量的参数化有强依赖性，也令这些结果的物理意义蒙上了疑问。不过这些例子也表明，约化密度矩阵的自洽性可以决定量子宇宙学模型中环境的定义本身，还可以作为场论系统中选择可观测量 (指针) 基矢的判据——这是退相干理论中一个重要问题。

Higgs Boson, Renormalization Group and Naturalness in Cosmology

希格斯玻色子、重整化群与宇宙学中的自然性

Higgs Inflation Model

希格斯暴胀模型

In the works considered in the preceding section, the nonminimally coupled inflaton scalar field was associated with one of the scalars arising in the Grand Unification Theories (GUT). Pioneering idea that the nonminimally coupled inflaton is nothing but the Higgs field of electroweak sector of the Standard Model was put forward in [44]. The Lagrangian of this model in the graviton-inflaton sector reads,

在前述章节讨论的研究中，非最小耦合的暴胀子标量场对应大统一理论 (GUT) 中产生的某一个标量场。非最小耦合暴胀子正是粒子物理标准模型电弱 sector 的希格斯场这一开创性观点由文献 [44] 提出。该模型中引力子-暴胀子部分的拉格朗日量如下:

$$L\left(g_{\mu\nu},\Phi\right)=\frac{1}{2}\left(M_{\rm P}^2+\xi|\Phi|^2\right)R-\frac{1}{2}|\nabla\Phi|^2-V\left(|\Phi|\right), \tag{33}$$

$$V\left(|\Phi|\right)=\frac{\lambda}{4}\left(|\Phi|^2-v^2\right)^2,\ |\Phi|^2=\Phi^\dagger\Phi, \tag{34}$$

where Φ is a scalar field multiplet, whose expectation value plays the role of an inflaton and which has a strong nonminimal curvature coupling with $\xi \gg 1$. Here, $M_{\rm P}=m_{\rm P}/\sqrt{8\pi}\approx 2.4\times 10^{18}$GeV is a reduced Planck mass, λ is a quartic self-coupling of Φ , and v is a symmetry breaking scale.

其中 Φ 是标量场多重态，其真空期望值扮演暴胀子的角色，并且和 $\xi \gg 1$ 存在强非最小曲率耦合。此处 $M_{\rm P} = m_{\rm P}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$GeV 是约化普朗克质量，$\lambda$ 是 Φ 的四次自耦合，v 是对称性破缺能标。

It was advocated in [44] that for the case when the scalar field is the Higgs field, the corresponding CMB data are consistent with the WMAP observations in the tree-level approximation of the theory. The further history of this nonminimally coupled Higgs inflation model was as follows. The methods of [35, 38, 80] were used to extend the predictions in this model to the one-loop level [45]. This has led immediately to the lower bound on the Higgs mass $M_{\rm H} \approx 230$GeV , originating from the observational restrictions on the CMB spectral index [45]. However, this conclusion did not take into account $O(1)$ effects of the renormalization group (RG) running, which qualitatively change the situation. This was nearly simultaneously observed in [48] and in [46], where the RG improvement of the one-loop results of [45] has decreased the lower bound on the Higgs mass to about 135 GeV.

文献 [44] 指出，当该标量场为希格斯场时，对应宇宙微波背景 (CMB) 数据在理论树级近似下与 WMAP 观测结果一致。这种非最小耦合希格斯暴胀模型的后续发展如下: 研究者利用 [35, 38, 80] 的方法将该模型的预言拓展到了单圈水平 [45]，立刻得到了希格斯质量 $M_{\rm H} \approx 230$GeV 的下限，该下限来自 CMB 谱指数的观测限制 [45]。但这一结论没有考虑重整化群 (RG) 跑动的 $O(1)$ 效应，而该效应会从定性上改变结论。这一点几乎同时被文献 [48] 和 [46] 发现，对 [45] 的单圈结果做 RG 改进后，希格斯质量的下限被降低到了约 135 GeV。

Quantitatively, this result was confirmed in [49], where we suggested the RG improvement of our one-loop results in [45] and found a range of the Higgs mass that is compatible with the CMB. Both the lower and upper boundary of this range were determined by the lower WMAP bound on the CMB spectral index - its value accepted at that time to be $n_s \approx 0.94$. The predictions of this model have also been extended to the two-loop approximation [47, 48], which has led to a reduction of the lower bound on the Higgs mass range by about 10GeV , which nearly coincides with the observed LHC value of 125 GeV.

这一定量结果在文献 [49] 中得到验证，作者对文献 [45] 的单圈结果做了 RG 改进，找到了与 CMB 观测相容的希格斯质量范围。该范围的上下限都由 WMAP 给出的 CMB 谱指数下限决定，当时公认该下限的取值为 $n_s \approx 0.94$ 。该模型的预言也被拓展到双圈近似 [47, 48]，结果使希格斯质量范围的下限降低了约 10GeV ，几乎与大型强子对撞机 (LHC) 观测到的 125 GeV 实验值一致。

Simultaneously with the papers advocating Higgs inflation, including in particular its supersymmetric extension [81-84], there arose a number of objections to this model. Apart from the strong assumption that no 'new physics' is present between electroweak (EW) and inflation scales, it was criticized that the predictions for Higgs inflation rely on the perturbation theory, which is only valid below the strong coupling scale. The reason for this criticism is that, for the flat space perturbation theory with a vanishing Higgs field background, this scale turns out to be $M_{\rm P}/\xi$, which is much lower than the inflation scale $M_{\rm P}/\sqrt{\xi}$ [53, 85, 86] , rendering the application of the perturbation theory questionable. Moreover, the multicomponent nature of the Higgs field leads to the impossibility of canonically normalizing all its components in the Einstein frame [85, 87, 88] - the parametrization heavily employed in [47, 48] .

在支持希格斯暴胀 (包括其超对称拓展 [81-84]) 的研究发表的同时，学界也对该模型提出了诸多反对意见。除了"电弱 (EW) 能标到暴胀能标之间不存在新物理"这一强假设之外, 批评还指出希格斯暴胀的预言依赖微扰论，而微扰论仅在强耦合能标以下成立。提出该批评的原因是，对于希格斯场背景为零的平直空间微扰论来说，该强耦合能标为 M_P/ξ，远低于暴胀能标 $M_P/\sqrt{\xi}$ [53, 85, 86]，因此微扰论的应用合理性存疑。此外，希格斯场的多分量性质导致无法在爱因斯坦框架 [85, 87, 88] 中对所有分量做正则归一化——而该参数化方式被广泛用于 [47, 48] 中。

In the approach of asymptotically safe gravity [89-92], one has also started to discuss the Higgs inflation model. There, however, one has not incorporated the model (33) with large ξ, but has exploited a rather miraculous numerical observation - a certain relation between the EW instability in the SM and the Planck scale [93,94] (The fixed point of the running coupling $\lambda(t)$ occurs very close to the Planck scale t_P with $\lambda(t_P) = 0$.). In spite of all the objections, the remarkable conformity of the LHC tests and the Higgs mass range compatible with the CMB data makes this model extremely attractive. In this section we shall discuss in some detail certain aspects of the model of the Higgs inflation, following essentially the paper [50].

在渐近安全引力方法 [89-92] 中，人们也已经开始讨论希格斯暴胀模型。但其中并未纳入大 ξ 对应的模型 (33)，而是利用了一个相当神奇的数值观测——标准模型 (SM) 中的电弱不稳定性与普朗克能标之间存在特定关系 [93,94](跑动耦合 $\lambda(t)$ 的不动点出现在非常接近普朗克能标 t_P 处，且满足 $\lambda(t_P) = 0$ 。)。尽管存在诸多反对意见，该模型与 LHC 实验结果惊人一致，且其希格斯质量范围与 CMB 数据相容，这些特点让它极具吸引力。本节我们将基本遵循文献 [50]，较详细地讨论希格斯暴胀模型的若干方面。

The usual understanding of non-renormalizable theories is that renormalization of higher-dimensional operators does not effect the renormalizable sector of low-dimensional operators, because the former ones are suppressed by powers of a cutoff - the Planck mass M_P [89]. Therefore, beta functions of the Standard Model sector are not expected to be modified by gravitons.

对于非重整化理论的通常认知是，高维算符的重整化不会影响低维算符的可重整化部分，因为前者会被截断 (即普朗克质量 M_P) 的幂次压低 [89]。因此，一般认为标准模型部分的 β 函数不会被引力子修改。

The situation with the nonminimal coupling is more subtle. Due to the mixing of the Higgs scalar field with the longitudinal part of gravity in the kinetic term of the Lagrangian (33), an obvious suppression of pure graviton loops by the effective Planck mass, $M_P^2 + \xi\varphi^2 \gg M_P^2$, proliferates for large ξ to the sector of the Higgs field, so that certain parts of the beta functions are strongly damped by a large ξ [48,95]. Therefore, a special combination of coupling constants **A** which we call anomalous scaling [29] becomes very small and lowers the CMB-compatible Higgs mass bound. The importance of this quantity follows from the fact observed in [6, 29, 35] that, due to large ξ, quantum effects and their CMB manifestation are universally determined by **A** . The nature of this quantity is as follows.

非最小耦合的情况更加微妙。由于拉格朗日量 (33) 的动能项中希格斯标量场与引力的纵向部分混合，纯引力子圈被有效普朗克质量 $M_P^2 + \xi\varphi^2 \gg M_P^2$ 的明显压低，会在大 ξ 下扩散到希格斯场部分，因此 β 函数的某些部分会被大 ξ 强烈阻尼 [48,95]。因此，我们称之为反常标度的特定耦合常数组合 **A** 会变得非常小，进而降低了与 CMB 相容的希格斯质量界限。这个量的重要性源于 [6, 29, 35] 中观测到的事实: 由于大 ξ，量子效应及其 CMB 表现由 **A** 普适地决定。这个量的性质如下。

Let the model contain in addition to (33) also a set of scalar fields χ , vector gauge bosons A_μ and spinors ψ , which have an interaction with Φ dictated by the local gauge invariance. If we denote by φ the inflaton - the only nonzero component of the mean value of Φ in the cosmological state - then the quantum effective action of the system takes a general form

假设该模型除 (33) 外，还包含一组标量场 χ 、矢量规范玻色子 A_μ 和旋量场 ψ ，它们与 Φ 的相互作用由局域规范不变性决定。若用 φ 表示暴胀子 (即宇宙学状态下 Φ 平均值唯一非零的分量)，则该系统的量子有效作用量具有以下通用形式

$$S\left[g_{\mu\nu},\varphi\right] = \int d^4x g^{1/2}\left(-V(\varphi) + U(\varphi)R\left(g_{\mu\nu}\right) - \frac{1}{2}G(\varphi)(\nabla\varphi)^2 + \ldots\right), \tag{35}$$

where $V(\varphi), U(\varphi)$ and $G(\varphi)$ are the coefficients of the derivative expansion, and we disregard the contribution of higher-derivative operators that are negligible in the slow-roll approximation of the inflation theory. In this approximation, the dominant quantum contribution to these coefficients comes from the heavy massive sector of the model. In particular, the masses of the physical particles and Goldstone modes $m(\varphi)$, generated by their quartic, gauge and Yukawa couplings with φ , give rise to the Coleman-Weinberg potential - the one-loop contribution to the effective potential V in (35). Since $m(\varphi) \sim \varphi$, for large φ this potential is given by the following sum of boson and fermion contributions:

其中 $V(\varphi), U(\varphi)$ 和 $G(\varphi)$ 是导数展开的系数，我们忽略了在暴胀理论的慢滚近似下可忽略的高阶导数算符贡献。在此近似下，这些系数的主要量子贡献来自模型的大质量部分。具体而言，由各粒子与 φ 的四次耦合、规范耦合和汤川耦合生成的物理粒子与戈德斯通模 $m(\varphi)$ 的质量，会产生科尔曼-温伯格势，即 (35) 中有效势 V 的单圈贡献。由于 $m(\varphi) \sim \varphi$ ，对于大 φ ，该势由玻色子和费米子贡献的以下和式给出:

$$V^{1-\mathrm{loop}}(\varphi) = \sum_{\mathrm{particles}} (\pm 1)\frac{m^4(\varphi)}{64\pi^2}\ln\frac{m^2(\varphi)}{\mu^2} = \frac{\lambda\mathbf{A}}{128\pi^2}\varphi^4\ln\frac{\varphi^2}{\mu^2} + \ldots \tag{36}$$

and thus determines the dimensionless coefficient $\mathbf{A}$ - the anomalous scaling associated with the normalization scale μ in (36). Moreover, for $\xi \gg 1$ it is mainly this quantity and the dominant quantum correction to $U(\varphi)$ [49],

因此它确定了无量纲系数 $\mathbf{A}$ ，即与 (36) 中归一化标度 μ 关联的反常标度。此外，对于 $\xi \gg 1$ ，主要就是这个量和对 $U(\varphi)$ 的主导量子修正 [49]

$$U^{1-\mathrm{loop}}(\varphi) = \frac{3\xi\lambda}{32\pi^2}\varphi^2\ln\frac{\varphi^2}{\mu^2} + \ldots, \tag{37}$$

which determine the quantum rolling force in the effective equation of the inflationary dynamics [35, 80] and which yield the parameters of the CMB generated during inflation [29].

决定了暴胀动力学有效方程 [35, 80] 中的量子慢滚力，并给出了暴胀期间生成的 CMB 参数 [29]。

Inflation and its CMB are easy to analyse in the Einstein frame of fields, denoted by $\hat{g}_{\mu\nu}, \hat{\varphi}$, in which the action $\hat{S}\left[\hat{g}_{\mu\nu},\hat{\varphi}\right] = S\left[g_{\mu\nu},\varphi\right]$ has a minimal coupling $\hat{U} = M_{\mathrm{P}}^2/2$, a canonically normalized inflaton field $\hat{G} = 1$, and the new inflaton potential $\hat{V} = M_{\mathrm{P}}^4 V(\varphi)/4U^2(\varphi)$ (The Einstein and Jordan frames are related by the equations $\hat{g}_{\mu\nu} = 2U(\varphi)\, g_{\mu\nu}/M_{\rm P}^2, (d\hat{\varphi}/d\varphi)^2 = M_{\rm P}^2\left(GU + 3U'^2\right)/2U^2$.). At the inflationary scale with $\varphi > M_{\rm P}/\sqrt{\xi} \gg v$ and $\xi \gg 1$, this potential reads

在标记为 $\hat{g}_{\mu\nu}, \hat{\varphi}$ 的爱因斯坦场框架中，暴胀及其宇宙微波背景很容易分析，其中作用量 $\hat{S}\left[\hat{g}_{\mu\nu}, \hat{\varphi}\right] = S\left[g_{\mu\nu}, \varphi\right]$ 具有最小耦合 $\hat{U} = M_{\rm P}^2/2$ 、正则归一化暴胀子场 $\hat{G} = 1$ 和新的暴胀势 $\hat{V} = M_{\rm P}^4 V(\varphi)/4U^2(\varphi)$ (爱因斯坦框架与约旦框架由方程 $\hat{g}_{\mu\nu} = 2U(\varphi)\, g_{\mu\nu}/M_{\rm P}^2, (d\hat{\varphi}/d\varphi)^2 = M_{\rm P}^2\left(GU + 3U'^2\right)/2U^2$ 关联。)。在满足 $\varphi > M_{\rm P}/\sqrt{\xi} \gg v$ 和 $\xi \gg 1$ 的暴胀能标下，该势可写为

$$\hat{V} = \frac{\lambda M_{\rm P}^4}{4\xi^2}\left(1 - \frac{2M_{\rm P}^2}{\xi\varphi^2} + \frac{A_I}{16\pi^2}\ln\frac{\varphi}{\mu}\right), \tag{38}$$

where the parameter $\mathbf{A_I}$ represents the anomalous scaling modified by the quantum correction to the nonminimal curvature coupling (37),

其中参数 $\mathbf{A_I}$ 代表非最小曲率耦合 (37) 经量子修正后得到的反常标度，

$$\mathbf{A_I} = \mathbf{A} - 12\lambda = \frac{3}{8\lambda}\left(2g^4 + \left(g^2 + g'^2\right)^2 - 16y_t^4\right) - 6\lambda. \tag{39}$$

This quantity - which we shall call inflationary anomalous scaling - enters the expressions for the slow-roll parameters,

我们将这个量称为暴胀反常标度，它会进入慢滚参数的表达式，

$$\hat{\varepsilon} \equiv \frac{M_{\rm P}^2}{2}\left(\frac{1}{\hat{V}}\frac{d\hat{V}}{d\hat{\varphi}}\right)^2, \ \hat{\eta} \equiv \frac{M_{\rm P}^2}{\hat{V}}\frac{d^2\hat{V}}{d\hat{\varphi}^2}, \tag{40}$$

and ultimately determines all the inflation characteristics. In particular, the smallness of $\hat{\varepsilon}$ yields the range of the inflationary stage $\varphi > \varphi_{\rm end}$, terminating at a value of $\hat{\varepsilon}$ which we choose to be $\hat{\varepsilon}_{\rm end} = 3/4$. Under the natural assumption that perturbation expansion is applicable for $\mathbf{A_I}/64\pi^2 \ll 1$, the inflaton value at the exit from inflation then equals $\varphi_{\rm end} \simeq 2M_{\rm P}/\sqrt{3\xi}$. The value of φ at the beginning of the inflation stage of duration N in units of the e-folding number then reads [29]

并最终决定所有暴胀特征。具体而言，$\hat{\varepsilon}$ 很小给出了暴胀阶段 $\varphi > \varphi_{\rm end}$ 的范围，暴胀在 $\hat{\varepsilon}$ 等于我们取定的 $\hat{\varepsilon}_{\rm end} = 3/4$ 时结束。在微扰展开适用于 $\mathbf{A_I}/64\pi^2 \ll 1$ 的自然假设下，暴胀退出时的暴胀子取值即为 $\varphi_{\rm end} \simeq 2M_{\rm P}/\sqrt{3\xi}$ 。那么，以 e 折叠数为单位、持续时长为 N 的暴胀阶段，其初始时刻 φ 的取值为 [29]

$$\varphi^2 = \frac{4N}{3}\frac{M_{\rm P}^2}{\xi}\frac{e^x - 1}{x}, \tag{41}$$

$$x \equiv \frac{NA_I}{48\pi^2}, \tag{42}$$

where the special parameter x directly involves the anomalous scaling $\mathbf{A_I}$.

其中特殊参数 x 直接包含了反常标度 $\mathbf{A_I}$ 。

This relation determines the Fourier power spectrum for the scalar metric perturbation $\zeta, \Delta_\zeta^2(k) \equiv \langle k^3 \zeta_{\mathbf{k}}^2 \rangle = \widehat{V}/24\pi^2 M_P^4 \hat{\varepsilon}$, where the right-hand side is taken at the first horizon crossing, $k = aH$, relating the comoving perturbation wavelength k^{-1} to the e-folding number N ,

该关系确定了标量度规微扰 $\zeta, \Delta_\zeta^2(k) \equiv \langle k^3 \zeta_{\mathbf{k}}^2 \rangle = \widehat{V}/24\pi^2 M_P^4 \hat{\varepsilon}$ 的傅里叶功率谱，式中右端取首次视界穿越时 $k = aH$ 的值，该点将共动微扰波长 k^{-1} 与 e 折叠数 N 关联起来,

$$\Delta_\zeta^2(k) = \frac{N^2}{72\pi^2} \frac{\lambda}{\xi^2} \left(\frac{e^x - 1}{x e^x} \right)^2. \tag{43}$$

The CMB spectral index $n_s \equiv 1 + d \ln \Delta_\zeta^2 / d \ln k = 1 - 6\hat{\varepsilon} + 2\hat{\eta}$ and the tensor to scalar ratio $r = 16\hat{\varepsilon}$ correspondingly read as (Note that for $|x| \ll 1$ these predictions coincide with those of the $f(R) = \left(M_P^2/2\right)\left(R + R^2/6M^2\right)$ inflationary model [96] with the scalaron mass $M = M_P\sqrt{\lambda}/\sqrt{3}\xi$ [97-99].)

CMB 谱指数 $n_s \equiv 1 + d \ln \Delta_\zeta^2 / d \ln k = 1 - 6\hat{\varepsilon} + 2\hat{\eta}$ 和张标比 $r = 16\hat{\varepsilon}$ 对应表示为 (注意，对 $|x| \ll 1$ 而言，这些预言与具有标量子质量 $M = M_P\sqrt{\lambda}/\sqrt{3}\xi$ 的 $f(R) = \left(M_P^2/2\right)\left(R + R^2/6M^2\right)$ 暴涨模型 [96] 的结果一致 [97-99]。)

$$n_s = 1 - \frac{2}{N} \frac{x}{e^x - 1}, \tag{44}$$

$$r = \frac{12}{N^2} \left(\frac{x e^x}{e^x - 1} \right)^2. \tag{45}$$

Therefore, with the spectral index constraint $0.948 < n_s(k_0) < 0.986$ (the combined WMAP+SPT+BAO+ H_0 data at the 2σ confidence level with the pivot point $k_0 = 0.002\text{Mpc}^{-1}$ corresponding to $N \simeq 60$), these relations immediately give the range $-12 < A_I < 14$ for the inflationary anomalous scaling [29].

因此，结合谱指数约束 $0.948 < n_s(k_0) < 0.986$ (WMAP+SPT+BAO+ H_0 的联合数据在 2σ 置信水平下，pivot 点 $k_0 = 0.002\text{Mpc}^{-1}$ 对应 $N \simeq 60$)，这些关系可直接得出暴涨反常标度的范围为 $-12 < A_I < 14$ [29]。

In the Standard Model $\mathbf{A}$ is expressed in terms of the masses of the heaviest particles $-W^\pm$ boson, Z boson and top quark,

在标准模型中，$\mathbf{A}$ 可以用最重粒子——$-W^\pm$ 玻色子、Z 玻色子和顶夸克——的质量表示,

$$m_W^2 = \frac{1}{4} g^2 \varphi^2, \ m_Z^2 = \frac{1}{4}\left(g^2 + g'^2\right)\varphi^2, \ m_t^2 = \frac{1}{2} y_t^2 \varphi^2, \tag{46}$$

and the mass of the three Goldstone modes $m_G^2 = V'(\varphi)/\varphi = \lambda\left(\varphi^2 - v^2\right) \simeq \lambda\varphi^2$. Here, g and g' are the $SU(2) \times U(1)$ gauge couplings, g_s is the $SU(3)$ strong coupling, and y_t is the Yukawa coupling for the top quark. At the inflation stage, the Goldstone mass-squared m_G^2 is non-vanishing, in contrast to its zero on-shell value in the EW vacuum [100]. Therefore, Eq. (36) gives the expression

以及三个戈德斯通模 $m_G^2 = V'(\varphi)/\varphi = \lambda\left(\varphi^2 - v^2\right) \simeq \lambda\varphi^2$ 的质量。此处，g 和 g' 是 $SU(2) \times U(1)$ 规范耦合，g_s 是 $SU(3)$ 强耦合，y_t 是顶夸克的汤川耦合。在暴涨阶段，戈德斯通质量平方 m_G^2 非零，这不同于它在电弱真空中的零在壳值 [100]。因此，式 (36) 给出表达式

$$\mathbf{A} = \frac{3}{8\lambda}\left(2g^4 + \left(g^2 + g'^2\right)^2 - 16y_t^4\right) + 6\lambda. \tag{47}$$

In the conventional range of the not yet observed Higgs mass 115GeV $\leq M_{\rm H} \leq$ 180 GeV (of the time of publication of [104]), this quantity was in the range $-48 < A < -20$, which contradicted the CMB range given above (though this contradiction was of $O(1)$ nature rather than of decimal orders of magnitude).

在 [104] 发表时，尚未观测到的希格斯质量的常规范围低于 180 GeV 115GeV $\leq M_{\rm H} \leq$，该量处于范围 $-48 < A < -20$ 内，与上述给出的 CMB 范围矛盾 (尽管这种矛盾是 $O(1)$ 量级的，而非数量级级别的矛盾)。

However, the RG running of coupling constants is strong enough and drives **A** to the CMB-compatible range at the inflation scale. Below we show that the formalism of [29] stays applicable but with the EW **A** replaced by the running $\mathbf{A}(t)$, where $t = \ln(\varphi/\mu)$ is the running scale of the renormalization group (RG) improvement of the effective potential [101].

然而，耦合常数的重整化群跑动足够强，会将 **A** 驱动到暴涨尺度下符合 CMB 的范围。下文我们将证明，[29] 的形式体系仍然适用，只需将电弱 **A** 替换为跑动 $\mathbf{A}(t)$，其中 $t = \ln(\varphi/\mu)$ 是有效势能重整化群 (RG) 改进的跑动标度 [101]。

RG Improvement

RG 改进

According to the Coleman-Weinberg technique [101], the one-loop RG-improved effective action has the form (35), with

根据 Coleman-Weinberg 方法 [101]，单圈 RG 改进有效作用量具有形式 (35)，其中

$$V(\varphi) = \frac{\lambda(t)}{4} Z^4(t)\varphi^4, \tag{48}$$

$$U(\varphi) = \frac{1}{2}\left(M_{\rm P}^2 + \xi(t) Z^2(t)\varphi^2\right), \tag{49}$$

$$G(\varphi) = Z^2(t). \tag{50}$$

Here, the running scale $t = \ln(\varphi/M_t)$ is normalized at the top quark mass $\mu = M_t$ (we denote physical [pole] masses by capital letters in contrast to running masses; see (46) above) (Application of the Coleman-Weinberg technique removes the ambiguity in the choice of the RG scale in cosmology - an issue discussed in [102].). The running couplings $\lambda(t)$, $\xi(t)$ and the field renormalization $Z(t)$ incorporate a summation of powers of logarithms and belong to the solution of the RG equations

此处，跑动标度 $t=\ln(\varphi/M_t)$ 在顶夸克质量 $\mu=M_t$ 处归一化 (我们将物理 [极点] 质量用大写字母表示，以区别于跑动质量；参见上文 (46))(应用 Coleman-Weinberg 方法消除了宇宙学中 RG 标度选择的歧义——这是文献 [102] 中讨论的问题。)。跑动耦合 $\lambda(t),\xi(t)$ 和场重整化 $Z(t)$ 包含了对数幂次求和，是 RG 方程的解

$$\frac{dg_i}{dt}=\beta_{g_i},\ \frac{dZ}{dt}=\gamma Z \tag{51}$$

for the full set of coupling constants $g_i=(\lambda,\xi,g,g',g_s,y_t)$ in the ’ heavy’ sector of the model with the corresponding beta functions β_{g_i} and the anomalous dimension γ of the Higgs field.

适用于模型“重”区中全套耦合常数 $g_i=(\lambda,\xi,g,g',g_s,y_t)$，对应贝塔函数为 β_{g_i}，希格斯场的反常维数为 γ。

An important subtlety for these β functions is the effect of the nonminimal curvature coupling of the Higgs field. For large ξ , the kinetic term of the tree-level action has a strong mixing between the graviton $h_{\mu\nu}$ and the quantum part of the Higgs field σ on the background φ . Symbolically, it has the structure

这些 β 函数的一个重要微妙之处在于希格斯场非最小曲率耦合的效应。对于大的 ξ，树图级作用量的动能项中，引力子 $h_{\mu\nu}$ 和希格斯场量子部分 σ 在背景 φ 下存在强烈混合。它的符号结构为

$$\left(M_{\rm P}^2+\xi^2\varphi^2\right)h\nabla\nabla h+\xi\varphi\sigma\nabla\nabla h+\sigma\bigtriangleup\sigma,$$

which yields a propagator whose elements are suppressed by a small $1/\xi$ -factor in all blocks of the 2×2 graviton-Higgs sector. For large $\varphi\gg M_{\rm P}/\sqrt{\xi}$, the suppression of pure graviton loops is, of course, obvious because the effective Planck mass squared strongly exceeds the Einstein one, $M_{\rm P}^2+\xi\varphi^2\gg M_{\rm P}^2$. Due to the mixing, this suppression proliferates to the full graviton-Higgs sector of the theory and yields the Higgs propagator $s(\varphi)/\left(\bigtriangleup-m_H^2\right)$, which contains the suppression factor $s(\varphi)$ given by

由此得到的传播子，其元在 2×2 引力子-希格斯扇区的所有区块中都被一个小的 $1/\xi$ 因子压低。对于大的 $\varphi\gg M_{\rm P}/\sqrt{\xi}$，纯引力子圈的压低当然显而易见，因为有效普朗克质量平方远大于爱因斯坦普朗克质量平方，即 $M_{\rm P}^2+\xi\varphi^2\gg M_{\rm P}^2$。由于混合，这种压低扩散到理论的整个引力子-希格斯扇区，得到希格斯传播子 $s(\varphi)/\left(\bigtriangleup-m_H^2\right)$，其包含的压低因子 $s(\varphi)$ 由下式给出

$$s(\varphi)=\frac{M_{\rm P}^2+\xi\varphi^2}{M_{\rm P}^2+(6\xi+1)\,\xi\varphi^2}. \tag{52}$$

This mechanism [35, 80, 95] modifies the beta functions of the SM sector [48] at high-energy scales because the factor $s(\varphi)$, which is close to one at the EW scale $v\ll M_{\rm P}/\xi$, is very small for $\varphi\gg M_{\rm P}/\sqrt{\xi}, s\simeq 1/6\xi$. Such a modification justifies, in fact, the extension beyond the scale $M_{\rm P}/\xi$ which is interpreted in [86, 103] as a natural validity cut-off of the theory (The smallness of this cut-off could be interpreted as an inefficiency of the RG analysis beyond the range of validity of the model. However, the cut-off $M_{\rm P}/\xi\ll M_{\rm P}$ of [86,103] applies to energies (momenta) of scattering processes in flat spacetime with a small EW value of φ . For the inflation stage on the background of a large φ , this cut-off gets modified due to the increase in the effective Planck mass $M_{\rm P}^2+\xi\varphi^2\gg M_{\rm P}^2$ (and the associated decrease of the s -factor (52) - resummation of terms treated otherwise as perturbations in [103]). Thus, the magnitude of the Higgs field at inflation is not really indicative of the violation of the physical cut-off bound.).

该机制 [35, 80, 95] 修正了高能标下标准模型扇区的贝塔函数 [48]，因为在电弱标度 $v \ll M_P/\xi$ 下接近 1 的因子 $s(\varphi)$，在 $\varphi \gg M_P/\sqrt{\xi}, s \simeq 1/6\xi$ 时变得非常小。事实上，这种修正证成了在标度 M_P/ξ 之外的延拓，而该标度在 [86, 103] 中被解释为理论的自然有效截断。(该截断很小可被解释为，RG 分析在模型有效范围之外是无效的。然而，文献 [86,103] 中的截断 $M_P/\xi \ll M_P$ 适用于 φ 取较小电弱值的平坦时空中散射过程的能量 (动量)。对于大 φ 背景下的暴胀阶段，由于有效普朗克质量 $M_P^2 + \xi\varphi^2 \gg M_P^2$ 增加 (以及相关的 s 因子减小，见式 (52)——对文献 [103] 中按扰动处理的项进行了重求和)，该截断会发生改变。因此，暴胀阶段希格斯场的大小并不真正代表违反了物理截断边界。)

There is another important subtlety with the modification of beta functions. Goldstone modes, in contrast to the Higgs particle, do not have a mixing with gravitons in the kinetic term [47]. Therefore, their contribution is not suppressed by the s -factor of the above type. Separation of Goldstone contributions from the Higgs contributions leads to the following modification of the one-loop beta functions essentially differing from those of [48]:

β 函数的修改存在另一项重要细节: 与希格斯粒子不同，戈德斯通模的动能项中不包含与引力子的混合项 [47]。因此，它们的贡献不会被上述类型的 s 因子压低。将戈德斯通贡献与希格斯贡献分离后，单圈 β 函数得到如下修改，这与文献 [48] 的结果存在本质差异:

$$\beta_\lambda = \frac{\lambda}{16\pi^2}\left(18s^2\lambda + \mathbf{A}(t)\right) - 4\gamma\lambda, \tag{53}$$

$$\beta_\xi = \frac{6\xi}{16\pi^2}\left(1 + s^2\right)\lambda - 2\gamma\xi, \tag{54}$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{2}{3}g'^2 - 8g_s^2 + \left(1 + \frac{s}{2}\right)y_t^2\right) - \gamma y_t, \tag{55}$$

$$\beta_g = -\frac{39 - s}{12}\frac{g^3}{16\pi^2}, \tag{56}$$

$$\beta_{g'} = \frac{81 + s}{12}\frac{g'^3}{16\pi^2}, \tag{57}$$

$$\beta_{g_s} = -\frac{7g_s^3}{16\pi^2}. \tag{58}$$

Here, the anomalous dimension γ of the Higgs field is given by the standard expression in the Landau gauge,

此处，希格斯场的反常维数 γ 由朗道规范下的标准表达式给出，

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9g^2}{4} + \frac{3g'^2}{4} - 3y_t^2\right), \tag{59}$$

the anomalous scaling $\mathbf{A}(t)$ is defined by (47), and we have retained only the leading terms in $\xi \gg 1$. It will be important in what follows that this anomalous scaling contains the Goldstone contribution 6λ , so that the full β_λ in (53) has a λ^2 -term unsuppressed by $s(\varphi)$ at large scale $t = \ln(\varphi/\mu)$.

反常标度 $\mathbf{A}(t)$ 由式 (47) 定义，我们仅保留了 $\xi \gg 1$ 中的领头项。下文将会说明，该反常标度包含戈德斯通贡献 6λ 这一点十分重要，因此式 (53) 中的完整 β_λ 含有一个在大标度 $t = \ln(\varphi/\mu)$ 下不会被 $s(\varphi)$ 压低的 λ^2 项。

The inflationary stage in units of Higgs field e-foldings is very short, which allows one to use an approximation linear in $\Delta t \equiv t - t_{\text{end}} = \ln(\varphi/\varphi_{\text{end}})$, where the initial data point is chosen at the end of inflation t_{end}. Therefore, for the beta functions (53) and (54) with $s \ll 1$, we have

以希格斯场 e 折叠数为单位的暴胀阶段非常短，因此可以采用 $\Delta t \equiv t - t_{\text{end}} = \ln(\varphi/\varphi_{\text{end}})$ 下的线性近似，初始数据点取在暴胀结束时刻 t_{end}。因此，对于含 $s \ll 1$ 的 β 函数 (53) 与 (54)，我们得到

$$\lambda(t) = \lambda_{\text{end}}\left(1 - 4\gamma_{\text{end}}\Delta t + \frac{\mathbf{A}(t_{\text{end}})}{16\pi^2}\Delta t\right), \tag{60}$$

$$\xi(t) = \xi_{\text{end}}\left(1 - 2\gamma_{\text{end}}\Delta t + \frac{6\lambda_{\text{end}}}{16\pi^2}\Delta t\right), \tag{61}$$

where $\lambda_{\text{end}}, \gamma_{\text{end}}, \xi_{\text{end}}$ are determined at t_{end}, and $\mathbf{A}_{\text{end}} = \mathbf{A}(t_{\text{end}})$ is the particular value of the running anomalous scaling (47) at the end of inflation.

其中 $\lambda_{\text{end}}, \gamma_{\text{end}}, \xi_{\text{end}}$ 在 t_{end} 处确定，$\mathbf{A}_{\text{end}} = \mathbf{A}(t_{\text{end}})$ 是跑动反常标度 (47) 在暴胀结束时的特定取值。

On the other hand, the RG improvement of the effective action (48)-(50) implies that this action coincides with the tree-level action for a new field $\tilde{\varphi} = Z(t)\varphi$ with running couplings as functions of $t = \ln(\varphi/\mu)$ (the running of $Z(t)$ is slow and affects only the multi-loop RG improvement). Then, in view of (48)-(49), the RG improved potential takes at the inflation stage the form of the one-loop potential (38) for the field φ with a particular choice of the normalization point $\mu = \varphi_{\text{end}}$ and all the couplings replaced by their running values taken at t_{end}. Therefore, the formalism of [29] can be directly applied to find the CMB parameters of the model, which now turn out to be determined by the running anomalous scaling $\mathbf{A_I}(t)$ taken at t_{end}.

另一方面，有效作用量 (48)-(50) 的 RG 改进意味着，该作用量与新场 $\tilde{\varphi} = Z(t)\varphi$ 的树图作用量一致，其中耦合随 $t = \ln(\varphi/\mu)$ 跑动 ($Z(t)$ 的跑动非常缓慢，仅影响多圈 RG 改进)。结合式 (48)-(49)，RG 改进势在暴胀阶段呈现为场 φ 的单圈势 (38)，其中正规化点 $\mu = \varphi_{\text{end}}$ 取特定值，所有耦合替换为它们在 t_{end} 处的跑動值。因此，文献 [29] 的形式体系可以直接用于求解模型的宇宙微波背景参数，最终这些参数由取值在 t_{end} 处的跑动反常标度 $\mathbf{A_I}(t)$ 决定。

In contrast to the inflationary stage, the post-inflationary running is very large and requires numerical simulation [49]. We fix the $t = 0$ initial conditions for the RG equations (51) at the top quark scale $M_t = 171\text{GeV}$. For the constants g, g' and g_s, they read [104]

与暴胀阶段不同，暴胀后耦合的跑动效应非常显著，需要数值模拟 [49]。我们在顶夸克能标 $M_t = 171\text{GeV}$ 处设定 RG 方程 (51) 的 $t = 0$ 初始条件。常数 g, g' 和 g_s 的初始条件为 [104]

$$g^2(0) = 0.4202, g'^2(0) = 0.1291, g_s^2(0) = 1.3460\,, \tag{62}$$

where $g^2(0)$ and $g'^2(0)$ are obtained by a simple one-loop RG flow from the conventional values of $\alpha(M_Z) \equiv g^2/4\pi = 0.0338, \alpha'(M_Z) \equiv g'^2/4\pi = 0.0102$ at M_Z -scale, and the value $g_s^2(0)$ at M_t is generated by the numerical program of [105]. The analytical algorithm of transition between different scales for g_s^2 was presented in [106-108]. For the Higgs self-interaction constant λ and for the Yukawa top quark interaction constant y_t , the initial conditions are determined by the pole mass matching scheme originally developed in [109, 110] and presented in the appendix of [111].

其中 $g^2(0)$ 和 $g'^2(0)$ 由 M_Z 标度下 $\alpha(M_Z) \equiv g^2/4\pi = 0.0338, \alpha'(M_Z) \equiv g'^2/4\pi = 0.0102$ 的常规值通过简单单圈重整化群流得到，M_t 处的 $g_s^2(0)$ 值由文献 [105] 的数值程序生成。文献 [106-108] 给出了 g_s^2 在不同标度间转换的解析算法。对于希格斯自相互作用常数 λ 和顶夸克汤川相互作用常数 y_t ，其初始条件由最初在文献 [109, 110] 中开发、并在文献 [111] 附录中给出的极点质量匹配方案确定。

The initial condition $\xi(0)$ follows from the CMB normalization (43), $\Delta_\xi^2 \simeq 2.5\times\ 10^{-9}$, at the pivot point $k_0 = 0.002\text{Mpc}^{-1}$, which we choose to correspond to $N \simeq 60$. This yields the following estimate for the ratio of coupling constants,

初始条件 $\xi(0)$ 由 pivot 点 $k_0 = 0.002\text{Mpc}^{-1}$ 处的 CMB 归一化 (43) $\Delta_\xi^2 \simeq 2.5\times\ 10^{-9}$ 导出，我们选取该点对应 $N \simeq 60$ 。由此可得耦合常数比值的如下估计:

$$\frac{1}{Z_{\text{in}}^2}\frac{\lambda_{\text{in}}}{\xi_{\text{in}}^2} \simeq 0.5\times 10^{-9}\left(\frac{x_{\text{in}}\exp x_{\text{in}}}{\exp x_{\text{in}} - 1}\right)^2, \tag{63}$$

at the moment of the first horizon crossing for $N = 60$, which we call the ’ beginning’ of inflation and label by $t_{\text{in}} = \ln(\varphi_{\text{in}}/M_t)$ with φ_{in} defined by (41). Thus, the RG equations (51) for the six couplings $(g, g', g_s, y_t, \lambda, \xi)$ with five initial conditions and the final condition for ξ uniquely determine the needed RG flow.

该估计对应 $N = 60$ 第一次出视界时刻，我们将该时刻称为暴胀的“开端”，记为 $t_{\text{in}} = \ln(\varphi_{\text{in}}/M_t)$ ，其中 φ_{in} 由式 (41) 定义。因此，针对六个耦合 $(g, g', g_s, y_t, \lambda, \xi)$ 的重整化群方程 (51)，结合五个初始条件和 ξ 的终态条件，可以唯一确定所需的重整化群流。

The RG flow covers also the inflationary stage from the chronological end of inflation t_{end} to t_{in} . At the end of inflation we choose the value of the slow roll parameter $\hat{\varepsilon} = 3/4$, and $\varphi_{\text{end}} \equiv M_t e^{t_{\text{end}}} \simeq M_{\text{P}}\sqrt{4/3\xi_{\text{end}}}$. Thus, the duration of inflation in units of inflaton field e-foldings $t_{\text{in}} - t_{\text{end}} = \ln(\varphi_{\text{in}}/\varphi_{\text{end}}) \simeq \ln N/2 \sim 2$ [49] is very short relative to the post-inflationary evolution $t_{\text{end}} \sim 35$. The approximation linear in the logarithms implies the bound $|\mathbf{A_I}(t_{\text{end}})|\,\Delta t/16\pi^2 \ll 1$, which in view of $\Delta t < t_{\text{in}} - t_{\text{end}} \simeq \ln N/2$ holds for $|\mathbf{A_I}(t_{\text{end}})|/16\pi^2 \ll 0.5$.

重整化群流也覆盖了从暴胀时间终点 t_{end} 到 t_{in} 的暴胀阶段。在暴胀终点我们选取慢滚参数 $\hat{\varepsilon} = 3/4$ 的值 $\varphi_{\text{end}} \equiv M_t e^{t_{\text{end}}} \simeq M_{\text{P}}\sqrt{4/3\xi_{\text{end}}}$ 。因此，以暴胀子场 e 折叠数 $t_{\text{in}} - t_{\text{end}} = \ln(\varphi_{\text{in}}/\varphi_{\text{end}}) \simeq \ln N/2 \sim 2$ [49] 为单位的暴胀持续时间，相比暴胀后演化 $t_{\text{end}} \sim 35$ 非常短。对数线性近似给出约束 $|\mathbf{A_I}(t_{\text{end}})|\,\Delta t/16\pi^2 \ll 1$ ，结合 $\Delta t < t_{\text{in}} - t_{\text{end}} \simeq \ln N/2$ 可知该约束对 $|\mathbf{A_I}(t_{\text{end}})|/16\pi^2 \ll 0.5$ 成立。

The running of $\mathbf{A}(t)$ depends strongly on the behaviour of $\lambda(t)$. For small Higgs masses, the usual RG flow in the SM leads to an instability of the EW vacuum caused by negative values of $\lambda(t)$ in a certain range of t [111,112]. The same happens in the presence of a nonminimal curvature coupling. The numerical solution for $\lambda(t)$ is shown in Fig. 1 for five values of the Higgs mass and the value of top quark mass $M_t = 171\text{GeV}$. The lowest one corresponds to the boundary of the instability window,

$\mathbf{A}(t)$ 的跑动强烈依赖于 $\lambda(t)$ 的性质。对于小质量希格斯玻色子，标准模型 (SM) 中常规的重整化群流会因 $\lambda(t)$ 在 t 的一定范围内取负值，引发电弱 (EW) 真空不稳定性 [111,112]。存在非最小曲率耦合时也会出现同样的情况。图 1 展示了五个希格斯质量取值、顶夸克质量取 $M_t = 171\text{GeV}$ 时 $\lambda(t)$ 的数值解。最小值对应不稳定性窗口的边界，

$$M_{\mathrm{H}}^{\mathrm{inst}} \simeq 134.27\text{GeV}\,, \tag{64}$$

for which $\lambda(t)$ bounces back to positive values after vanishing at $t_{\text{inst}} \sim 41.6$ or $\varphi_{\text{inst}} \sim 80M_{\mathrm{P}}$. The shape of the corresponding effective potential in the Einstein frame is depicted in Fig. 2 and shows the existence of a false vacuum at this instability scale. It turns out that the relevant $\xi(t)$ is nearly constant and is about 5000 (see below), so that the factor (52) at t_{inst} is very small, $s \simeq 1/6\xi \sim 0.00005$. Thus, the situation is different from the usual SM with $s = 1$, and numerically, the critical value turns out to be higher than the known SM stability bound $\sim 125\text{GeV}$ [111].

此时 $\lambda(t)$ 在 $t_{\text{inst}} \sim 41.6$ 或 $\varphi_{\text{inst}} \sim 80M_{\mathrm{P}}$ 处归零后反弹回正值。爱因斯坦框架下对应有效势的形态如图 2 所示，可见该不稳定性标度处存在伪真空。计算表明相关的 $\xi(t)$ 近似为常数，约等于 5000(见下文)，因此 (52) 式在 t_{inst} 处的因子非常小，即 $s \simeq 1/6\xi \sim 0.00005$ 。因此，这一情况和存在 $s = 1$ 的常规标准模型不同，数值上临界值高于已知的标准模型稳定性界 $\sim 125\text{GeV}$ [111]。

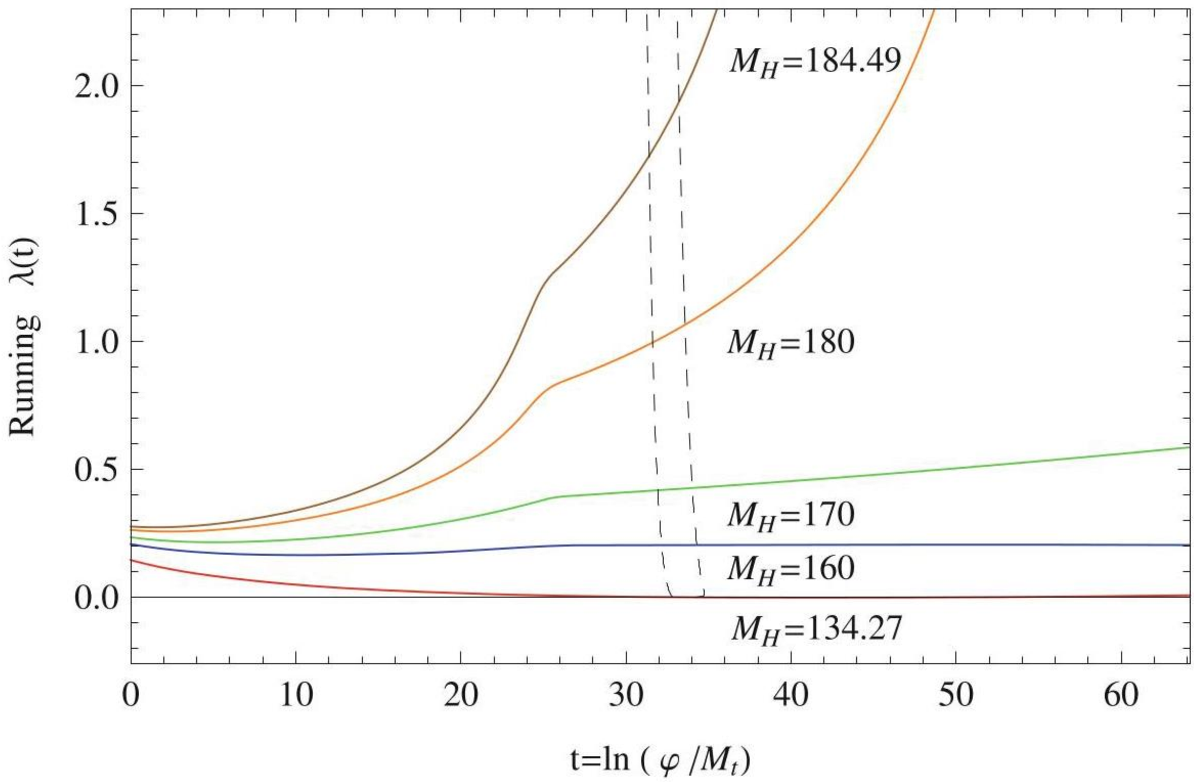


Fig. 1 Running $\lambda(t)$ for five values of the Higgs mass above the instability threshold. Dashed curves mark the boundaries of the inflation domain $t_{\text{end}} \leq t \leq t_{\text{in}}$ [49]

图 1 不稳定性阈值以上五个希格斯质量取值对应的 $\lambda(t)$ 跑动。虚线标记了暴涨区域 $t_{\text{end}} \leq t \leq t_{\text{in}}$ 的边界 [49]

Figure 1 shows that near the instability threshold $M_{\text{H}} = M_{\text{H}}^{\text{inst}}$, the running coupling $\lambda(t)$ stays very small for all scales t relevant to the observable CMB. This follows from the fact that the positive running of $\lambda(t)$ caused by the term $\left(18s^2+6\right)\lambda^2$ in β_λ ,(see (53)), is much slower for $s \ll 1$ than that of the usual SM driven by the term $24\lambda^2$.

图 1 显示，在不稳定性阈值 $M_{\text{H}} = M_{\text{H}}^{\text{inst}}$ 附近，对于可观测宇宙微波背景 (CMB) 相关的所有标度 t ，跑动耦合 $\lambda(t)$ 始终保持非常小的取值。这是因为，β_λ 中 $\left(18s^2+6\right)\lambda^2$ 项引发的 $\lambda(t)$ 正跑动，在 $s \ll 1$ 情形下远慢于 $24\lambda^2$ 项驱动的常规标准模型跑动 (见 (53) 式)。

The RG running of $\mathbf{A_I}(t)$ explains the main difference from the results of the one-loop calculations in [29]. $\mathbf{A_I}(t)$ runs from big negative values $\mathbf{A_I}(0) < -20$ at the EW scale to small but also negative values at the inflation scale below t_{inst} . This makes the CMB data compatible with the generally accepted Higgs mass range. Indeed, the knowledge of the RG flow immediately allows one to obtain $\mathbf{A_I}(t_{\text{end}})$ and x_{end} and thus to find the parameters of the CMB power spectrum (44)-(45) as functions of M_{H} . The parameter of primary interest - the spectral index - is given by (44) with $x = x_{\text{end}} \equiv N\mathbf{A_I}(t_{\text{end}})/48\pi^2$ and depicted in Fig. 3. Even for low values of the Higgs mass above the stability bound, n_s falls into the range admissible by the CMB constraint.

$\mathbf{A_I}(t)$ 的重整化群跑动解释了本文结果与文献 [29] 中单圈计算结果的主要差异。$\mathbf{A_I}(t)$ 从电弱标度处的大负值 $\mathbf{A_I}(0) < -20$, 跑动到低于 t_{inst} 的暴涨标度处的小负值。这使得 CMB 数据与公认的希格斯质量范围相容。实际上，了解重整化群流后可以直接得到 $\mathbf{A_I}(t_{\text{end}})$ 和 x_{end} ，进而得到 CMB 功率谱 (44)-(45), 它是 M_{H} 的函数。我们最关注的参数——谱指数——由代入 $x = x_{\text{end}} \equiv N\mathbf{A_I}(t_{\text{end}})/48\pi^2$ 的 (44) 式给出，结果绘制在图 3 中。即便是高于稳定性界的低质量希格斯玻色子，n_s 也落在 CMB 约束允许的范围内。

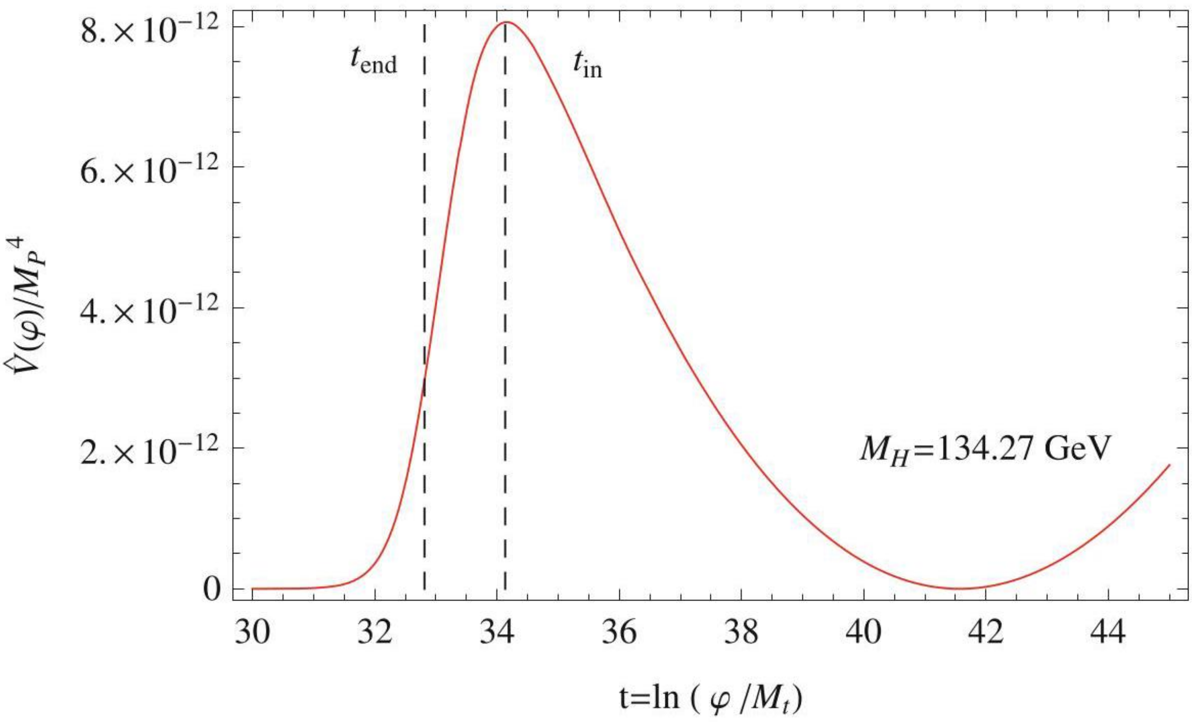


Fig. 2 The Einstein frame effective potential for the instability threshold $M_{\rm H}^{\rm inst} = 134.27{\rm GeV}$. A false vacuum occurs at the instability scale $t_{\rm inst} \simeq 41.6, \varphi_{\rm inst} \sim 80 M_{\rm P}$, which is much higher than the Planck scale. A possible domain of inflation (ruled out by the lower n_s CMB bound) is again marked by dashed lines [49]

图 2 不稳定性阈值 $M_{\rm H}^{\rm inst} = 134.27{\rm GeV}$ 的爱因斯坦框架有效势。虚真空出现在不稳定性标度 $t_{\rm inst} \simeq 41.6, \varphi_{\rm inst} \sim 80 M_{\rm P}$ 处，远高于普朗克标度。可能的暴胀区域 (被 CMB 下限 n_s 排除) 再次用虚线标出 [49]

The spectral index drops below 0.95 only for large $x_{\rm end} < 0$ or large negative $\mathbf{A_I}(t_{\rm end})$, which happens only when $M_{\rm H}$ either approaches the instability bound or exceeds 180GeV at the decreasing branch of the n_s graph. Thus, we get lower and upper bounds on the Higgs mass, which both follow from the lower bound of the CMB data. Numerical analysis for the corresponding $x_{\rm end} \simeq -1.4$ gives for $M_t = 171{\rm GeV}$ the following range for the CMB-compatible Higgs mass:

仅当 $x_{\rm end} < 0$ 较大或 $\mathbf{A_I}(t_{\rm end})$ 负取值较大时，谱指数才会低于 0.95，这种情况仅当 $M_{\rm H}$ 接近不稳定性边界，或在 n_s 图的递减分支上超过 180GeV 时发生。因此我们得到希格斯质量的上下限，二者均由 CMB 数据的下限导出。对对应 $x_{\rm end} \simeq -1.4$ 的数值分析给出，与 CMB 相容的希格斯质量 $M_t = 171{\rm GeV}$ 范围如下:

$$135.6{\rm GeV} \le M_{\rm H} \le 184.5{\rm GeV} . \tag{65}$$

Both bounds belong to the nonlinear domain of (44) with $x_{\rm end} \simeq -1.4$, but the quantity $|\mathbf{A_I}(t_{\rm end})|/16\pi^2 = 0.07 \ll 0.5$ satisfies the restriction mentioned above, and their calculation is still in the domain of our log-linear approximation.

两个边界都属于 (44) 式含 $x_{\text{end}} \simeq -1.4$ 的非线性区域，但物理量 $|\mathbf{A_I}(t_{\text{end}})|/16\pi^2 = 0.07 \ll 0.5$ 满足上述限制，且它们的计算仍在我们的对数线性近似范围内。

The upper bound on n_s does not generate restrictions on M_{H} . The above bounds were obtained for $M_t = 171\text{GeV}$. Results for the neighbouring values $M_t = 171 \pm 2\text{GeV}$ are presented in Fig. 3 to show the pattern of their dependence on M_t .

n_s 的上限不会对 M_{H} 产生限制。上述边界是在 $M_t = 171\text{GeV}$ 条件下得到的。图 3 给出了相邻取值 $M_t = 171 \pm 2\text{GeV}$ 的结果，以展示它们对 M_t 的依赖关系。

The expression (35) is a truncation of the curvature and derivative expansion of the full effective action. It was repeatedly claimed that with large ξ the weak field version of this expansion on a flat (and empty) space background has a cutoff $4\pi M_{\text{P}}/\xi$ [86, 103] . This scale is essentially lower than the Higgs field during inflation $\varphi \sim M_{\text{P}}/\sqrt{\xi}$ and, therefore, seems to invalidate predictions based on (35) unless an unnatural suppression of higher-dimensional operators is assumed.

表达式 (35) 是全有效作用量曲率与导数展开的截断形式。已有研究反复指出，当 ξ 很大时，该展开在平直 (空) 空间背景下的弱场版本存在截断标度 $4\pi M_{\text{P}}/\xi$ [86, 103] 。该标度远低于暴胀过程中希格斯场的对应标度 $\varphi \sim M_{\text{P}}/\sqrt{\xi}$ ，因此除非假定高维算符存在非自然压低，否则 (35) 得到的预测似乎不成立。

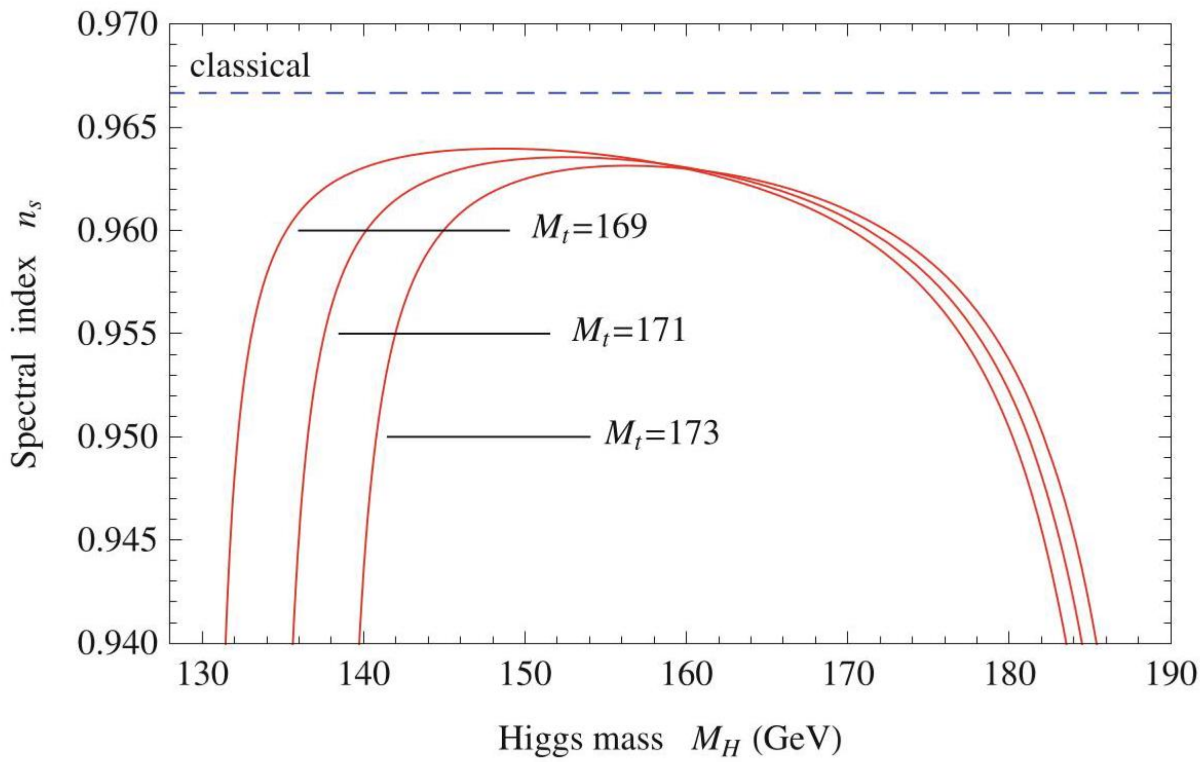


Fig. 3 The spectral index n_s as a function of the Higgs mass M_{H} for three values of the top quark mass [49]

图 3 三种顶夸克质量取值下，谱指数 n_s 随希格斯质量 $M_{\rm H}$ 变化的函数关系 [49]

The attempt to improve the situation by transition to the Einstein frame [113] was claimed to fail [85, 87, 88] for a multiplet Higgs field involving Nambu-Goldstone modes.

有研究指出，通过转换到爱因斯坦框架改善该问题的尝试 [85, 87, 88] 对包含南部-戈德斯通模的多重希格斯场不成立。

In the following, we show that these objections against naturalness are not conclusive. First, as mentioned above, a large value of φ during inflation is not really indicative of a large physical scale of the problem. In contrast to curvature and energy density, the inflaton itself is not a physical observable, but rather a configuration space coordinate of the model. Second, we now show that the inflation scale actually lies below the gradient expansion cut-off, and this justifies the naturalness of the obtained results. No transition to another conformal frame is needed for this purpose, but rather a resummation accounting for a transition to a large φ background.

下文我们将说明，这些针对自然性的反对意见并不具有决定性。首先，如上所述，暴胀过程中 φ 取大值并不代表该问题存在大物理标度。与曲率和能量密度不同，暴胀子本身并非物理可观测量，而是模型的位形空间坐标。其次，我们现在说明，暴胀标度实际上低于梯度展开截断，这证明了我们所得结果的自然性。为此无需转换到其他共形框架，只需要通过重求和处理转换到大背景 φ 的情况。

Indeed, the main peculiarity of the model (33) is that in the background field method with small derivatives, the role of the effective Planck mass is played by $\sqrt{M_{\rm P}^2+\xi\varphi^2}$. The power-counting method of [103] underlying the derivation of the cut-off $4\pi M_{\rm P}/\xi$ also applies here, but with the Planck mass $M_{\rm P}$ replaced by the effective one, $M_{\rm P}\to\sqrt{M_{\rm P}^2+\xi\varphi^2}>\sqrt{\xi}\varphi$. The resulting cut-off is thus bounded from below by

事实上，模型 (33) 的主要特殊之处在于，在小导数背景场方法中，有效普朗克质量的角色由 $\sqrt{M_{\rm P}^2+\xi\varphi^2}$ 承担。导出截断 $4\pi M_{\rm P}/\xi$ 所依据的文献 [103] 中的幂次计数方法在这里同样适用，只是普朗克质量 $M_{\rm P}$ 被替换为有效普朗克质量 $M_{\rm P}\to\sqrt{M_{\rm P}^2+\xi\varphi^2}>\sqrt{\xi}\varphi$ 。因此，所得截断的下界为

$$\Lambda\left(\varphi\right)=\frac{4\pi\varphi}{\sqrt{\xi}}, \tag{66}$$

and this bound can be used as a running cut-off of the gradient and curvature expansion. The origin of this cut-off can be demonstrated in the one-loop approximation. When calculated in the Jordan frame, for the one-loop divergences quadratic in the curvature R , the dominating ξ contribution is (this can be easily deduced from Appendix of [49])

且该界限可作为梯度和曲率展开的跑动截断使用。该截断的起源可在单圈近似下得到证明。在约旦框架下计算时，对于曲率二次项的单圈发散 R ，主导 ξ 贡献为 (这可从文献 [49] 的附录中轻松推导出)

$$\xi^2\frac{R^2}{16\pi^2}\,. \tag{67}$$

As compared to the tree-level part linear in the curvature $\sim \left(M_{\rm P}^2+\xi\varphi^2\right)R$, the one-loop R^2 -term turns out to be suppressed by the above cut-off factor $16\pi^2\left(M_{\rm P}^2+\ \ \xi\varphi^2\right)/\xi^2\simeq\Lambda^2$.

与曲率中为线性的树图项 $\sim \left(M_{\rm P}^2+\xi\varphi^2\right)R$ 相比，单圈 R^2 项被上述截断因子 $16\pi^2\left(M_{\rm P}^2+\ \ \xi\varphi^2\right)/\xi^2\simeq\Lambda^2$ 抑制。

The on-shell curvature estimate at the inflation stage reads $R\sim V/U\sim\lambda\varphi^2/\xi$ in the Jordan frame, so that the resulting curvature expansion runs in powers of

膨胀阶段的在壳曲率估计在约旦框架中表示为 $R\sim V/U\sim\lambda\varphi^2/\xi$ ，因此最终曲率展开是按幂次进行的

$$\frac{R}{\Lambda^2}\sim\frac{\lambda}{16\pi^2}\tag{68}$$

and remains valid in the usual perturbation theory range of SM, for which $\lambda/16\pi^2\ll 1$. This works perfectly well in our Higgs inflation model, because in the full CMB-compatible range of the Higgs mass, one has $\lambda<2$ (see Fig. 1).

且在标准模型的常规微扰论范围内仍然有效，该范围满足 $\lambda/16\pi^2\ll 1$ 。这在我们的希格斯暴胀模型中完全成立，因为在与宇宙微波背景相容的整个希格斯质量范围内，都满足 $\lambda<2$ (参见图 1)。

From the viewpoint of the gradient expansion for φ , this cut-off is even more efficient. Indeed, the inflaton field gradient can be expressed in terms of the inflaton potential $\hat{V}$ and the inflation smallness parameter $\hat{\varepsilon}$ taken in the Einstein frame, $\dot{\varphi}\simeq\ \left(\varphi^2/M_{\rm P}^2\right)\left(\xi\hat{\varepsilon}\hat{V}/18\right)^{1/2}$. With $\hat{V}\simeq\lambda M_{\rm P}^4/4\xi^2$, this immediately yields the gradient expansion in powers of

从 φ 梯度展开的角度来看，这个截断的效率更高。事实上，暴胀子场梯度可以用爱因斯坦框架下的暴胀势 $\hat{V}$ 和暴胀小参数 $\hat{\varepsilon}$ 表示，即 $\dot{\varphi}\simeq\ \left(\varphi^2/M_{\rm P}^2\right)\left(\xi\hat{\varepsilon}\hat{V}/18\right)^{1/2}$ 。引入 $\hat{V}\simeq\lambda M_{\rm P}^4/4\xi^2$ 后，可直接得到按幂次展开的梯度展开式

$$\frac{\partial}{\Lambda}\sim\frac{1}{\Lambda}\frac{\dot{\varphi}}{\varphi}\simeq\frac{\sqrt{\lambda}}{48\pi}\sqrt{2\hat{\varepsilon}},\tag{69}$$

which is even better than (68) by a factor ranging from $1/N$ at the beginning of inflation to $O(1)$ at the end of it.

这比式 (68) 的效果更好，因子范围从暴胀初期的 $1/N$ 到暴胀末期的 $O(1)$ 。

Equations (68) and (69) justify the effective action truncation in (35) in the inflationary domain. Thus, only multi-loop corrections to the coefficient functions $V(\varphi)$, $U(\varphi)$ and $G(\varphi)$ may stay beyond control in the form of higher-dimensional operators $(\varphi/\Lambda)^n$ and violate the flatness of the effective potential necessary for inflation. However, in view of the form of the running cut-off (66), they might be large, but do not affect the shape of these coefficient functions because of the field independence of the ratio φ/Λ . Only the logarithmic running of couplings in (48)- (50) controlled by the RG dominates the quantum input in the inflationary dynamics and its CMB spectra (This is like the logarithmic term in (38), which dominates over the nearly flat classical part of the inflaton potential and qualitatively modifies the tree-level predictions of the theory [45].).

式 (68) 和 (69) 证明了暴胀区域中式 (35) 的有效作用量截断是合理的。因此，仅系数函数 $V(\varphi)$, $U(\varphi)$ 和 $G(\varphi)$ 的多圈修正可能以高维算符 $(\varphi/\Lambda)^n$ 的形式保留在控制范围之外，破坏暴胀所需的有效势平坦性。但考虑到跑动截断 (66) 的形式，这类修正可能很大，不过由于比值 φ/Λ 与场无关，它们不会影响这些系数函数的形状。只有 (48)-(50) 中由 RG 控制的耦合常数对数跑动，才主导暴胀动力学及其 CMB 谱中的量子贡献 (这类似 (38) 中的对数项，它主导暴胀子势中近乎平坦的经典部分，从定性上改变了理论的树级预言 [45])。

Before summing up, let us formulate once again the basic assumptions, made in [45, 49, 50] . The relation between the observable cosmological data (the spectral index n_S) and the data coming from particle physics was established. This relation arises due to the fact that in the early Universe, the classical Friedmann evolution is essentially modified by quantum corrections. These quantum corrections depend on interaction couplings of Standard Model particles with the Higgs field, playing in the model under consideration the role of inflaton. To relate measures at the electroweak scale values of these couplings with their hypothetical values at the inflationary scale, we have used the renormalization group formalism. We did it in spite of the well-known non-renormalizability of quantum gravity, using two facts. First, below a certain scale one can use an effective field theory and all the participants of the related discussions agree that this scale is not lower than m_P/ξ . Second, for large values of the scalar field (or, in other words, at high energies), the theory possesses a scale invariance. It is this invariance which defends us from an uncontrollable growth of quantum corrections. The question arises: is not the transition between these two ”safe” regions of values of the scalar field dangerous? The hypothesis which we used consists in the hope that the use of the continuous s -factor constructs for us some kind of bridge between these two regions, smoothly interpolating between low values of the Higgs field, where the effective theory is valid and high values, where the almost exact scale invariance is present.

在总结之前，让我们再次阐明 [45, 49, 50] 中作出的基本假设。我们已经建立了可观测宇宙学数据 (谱指数 n_S) 与粒子物理给出的数据之间的关联。这一关联的来源是: 在早期宇宙中，经典弗里德曼演化发生了本质上被量子修正改变的情况。这些量子修正依赖于标准模型粒子与希格斯场的相互作用耦合，在所研究模型中希格斯场扮演暴胀子的角色。为了将这些耦合在电弱标度的测量值与它们在暴胀标度的假设值联系起来，我们使用了重整化群形式体系。尽管量子引力众所周知是不可重整的，我们依然借助两个事实完成了这一工作。首先，在某一特定标度以下，我们可以使用有效场理论，所有参与相关讨论的研究者都同意该标度不低于 m_P/ξ 。其次，当标量场取值很大 (换言之，在高能量下) 时，该理论具有标度不变性。正是这种不变性避免了量子修正出现不受控的增长。随之而来的问题是: 这两个标量场取值“安全”区域之间的过渡是否存在危险？我们采用的假设是: 我们相信使用连续的 s 因子能为我们在这两个区域之间搭建起某种桥梁，在有效理论成立的希格斯场低取值区和近似精确标度不变性存在的高取值区之间完成平滑插值。

Palatini Version of Higgs Inflation Model

希格斯暴涨模型的帕拉蒂尼版本

The nonminimal Higgs inflation model had a quite intensive and somewhat unexpected development in the years following the discovery of Higgs boson. Indeed, some old ideas put forward at the dawn of general relativity were applied in a quite new context. This is the so-called Palatini variational method [114-116]. In the metric approach one has a relation between the affine connection and spacetime metric postulating that the covariant derivative of the latter is vanishing. If torsion tensor is zero, one obtains the Levi-Civita connection defined by Christoffel symbols. One can instead consider the affine connection and metric as independent variables and try to obtain the relation between them starting from the variational principle. The Hilbert-Einstein action in this case can be obtained as follows: one defines the Riemann-Christoffel curvature tensor and Ricci tensor using only the affine connection, whereas for the construction of the Ricci scalar, one needs to introduce the metric. Then, varying thus the defined Hilbert-Einstein action with respect to the affine connection coefficients, one obtains the equation giving the standard Levi-Civita relation between the metric and connection, while the metric variation gives Einstein equations.

在希格斯玻色子发现后的数年间，非最小耦合希格斯暴涨模型得到了相当深入且有些出乎意料的发展。实际上，广义相对论萌芽时期提出的一些旧思想被应用在了一个全新的语境中，这就是所谓的帕拉蒂尼变分法 [114-116]。在度规方法中，仿射联络与时空度规之间满足协变导数为零的关系；若挠率张量为零，就能得到由克里斯托费尔符号定义的列维-奇维塔联络。我们也可以将仿射联络和度规视为独立变量，从变分原理出发推导二者的关系。这种情况下的希尔伯特-爱因斯坦作用量可以按如下方式得到: 仅用仿射联络定义黎曼-克里斯托费尔曲率张量和里奇张量，而构造里奇标量时需要引入度规。之后，对这样定义的希尔伯特-爱因斯坦作用量关于仿射联络系数变分，就能得到度量和联络之间标准的列维-奇维塔关系，而对度规变分则给出爱因斯坦方程。

All this is true in the absence of a nontrivial coupling between matter fields and the quantities involving the affine connection. In the presence of nonminimal coupling between the Ricci scalar and the scalar field, the connection variation gives for the latter the expression containing beside the standard Christoffel part extra terms depending on the scalar field. Indeed, if the Lagrangian of the model includes the nonminimal coupling term

以上结论在物质场与包含仿射联络的量之间不存在非平凡耦合时成立。当里奇标量和标量场之间存在非最小耦合时，对联络变分得到的结果中，除了标准的克里斯托费尔部分外，还会包含依赖于标量场的额外项。实际上，如果模型的拉格朗日量包含非最小耦合项

$$L(g,\Gamma,\phi) = \sqrt{-g}U(\phi)\,g^{\mu\nu}R_{\mu\nu}(\Gamma)\,, \tag{70}$$

then its variation with respect to the affine connection $\Gamma^{\alpha}_{\mu\nu}$ gives the expression containing ϕ and its spacetime derivatives,

那么它对仿射连接 $\Gamma^{\alpha}_{\mu\nu}$ 的变分就会得到包含 ϕ 及其时空导数的表达式,

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}\right) + \frac{1}{2}\frac{U'(\phi)}{U(\phi)}\left(\delta^{\alpha}_{\mu}\phi_{,\nu} + \delta^{\alpha}_{\nu}\phi_{,\mu} - g_{\mu\nu}g^{\alpha\beta}\phi_{,\beta}\right).$$

(71)

Here the prime denotes the derivative with respect to the scalar field ϕ . For a constant $U(\phi)$ additional terms in the right-hand side of this equation vanish and it acquires a standard metric form. For nonminimal coupling the components of the Ricci tensor and scalar get nontrivial dependence on the scalar field and its spacetime derivatives, and the dynamics of the model changes significantly.

此处撇号表示对标量场 ϕ 求导。当 $U(\phi)$ 为常数时，该方程右侧的额外项会消失，方程退化为标准度规形式。对于非最小耦合，里奇张量和里奇标量会对标量场及其时空导数产生非平凡依赖，模型的动力学也会发生显著改变。

These effects were studied in detail for the nonminimal Higgs model in the paper [117]. It was found that the two formalisms differ in their predictions for various cosmological parameters. The main reason is that the dependence on the nonminimal coupling parameter is very different in the two formalisms. For successful inflation, the Palatini approach prefers a much larger value of the nonminimal coupling than the metric approach. Unlike in metric formalism, in the Palatini version, inflaton stays well below the Planck scale thereby providing a natural inflationary epoch. In the subsequent paper [118] special attention was paid to the question of unitarity. It was stated that Higgs inflation does not suffer from unitarity violation since the UV cut-off lies parametrically much higher than the Hubble rate so that the unknown UV physics does not disrupt the inflationary dynamics. Higgs-Palatini inflation turns out to be UV-safe and minimal and endowed with predictive power.

文献 [117] 对非最小希格斯模型的这些效应进行了详细研究，结果表明两种形式体系对各类宇宙学参数的预测存在差异，核心原因是两种形式体系中，非最小耦合参数的依赖关系完全不同。为了实现成功暴涨，帕拉蒂尼方法要求非最小耦合的取值远大于度规方法。与度规形式体系不同，帕拉蒂尼版本中，暴涨子始终远低于普朗克能标，因此可以自然地产生暴涨阶段。后续文献 [118] 特别关注了幺正性问题，指出希格斯暴涨不会破坏幺正性，因为紫外截断在参数上远高于哈勃率，未知的紫外物理不会破坏暴涨动力学。帕拉蒂尼希格斯暴涨被证明是紫外安全、最小化且具备预测能力的模型。

The nonminimal Higgs inflation in the Palatini formalism with loop corrections was treated in [119]. It was shown that the observable cosmological parameters are different in the Palatini approach and in the metric one. For example, the tensor to scalar ratio in the Palatini formalism is much lower than in the metric one. Thus, future observations can give indications concerning the nature of affine connection in our real physical Universe. In [120] the Coleman-Weinberg potential was studied for the nonminimal Higgs model within Palatini and metric formalisms and it was shown that these two formalisms predict different e -folding numbers. A wide class of scalar-tensor models of inflation in both formalisms was also considered in [121]. In particular, it was shown that for a simple Galileon model inflation naturally arises within Palatini formalism.

文献 [119] 研究了帕拉蒂尼形式体系中包含圈修正的非最小希格斯暴涨，结果表明帕拉蒂尼方法和度规方法得到的可观测宇宙学参数存在差异。例如，帕拉蒂尼形式体系中的张标比远低于度规形式体系。因此，未来的观测可以为我们真实物理宇宙中仿射联络的本质提供线索。文献 [120] 研究了帕拉蒂尼和度规两种形式体系下非最小希格斯模型的科尔曼-温伯格势，结果表明两种形式体系预测出不同的 e 暴涨 e 折数。文献 [121] 还研究了两种形式体系下广泛的一类标量-张量暴涨模型，特别指出对于简单的伽利略模型，帕拉蒂尼形式体系可以自然地产生暴涨。

Quantum corrections in the Palatini version of the nonminimal Higgs model were studied and compared with the analogous effects in the metric theory in [122]. In particular, interesting relations between the bounds on top quark mass and the bounds on the parameter of the nonminimal coupling were discovered.

文献 [122] 研究了非最小希格斯模型帕拉蒂尼版本中的量子修正，并与度规理论中的对应效应进行了对比，特别发现了顶夸克质量界限和非最小耦合参数界限之间存在有趣的关联。

Finally, questions concerning different parametrizations of the Higgs doublets in the Palatini and metric formalisms were studied in the recent paper [123]. The Einstein-Cartan gravity with a non-vanishing torsion tensor was applied to Higgs inflation in [124]. Numerous parameters of such a model make its comparison with observations much richer. On the other hand, the Einstein-Cartan theory is the simplest natural extension of the Palatini approach to gravity theory and, as stated in [124], there are no reasons to exclude it from the family of possible candidates on the role of a fundamental theory of gravity.

最后，最近的文献 [123] 研究了帕拉蒂尼形式与度规形式中希格斯二重态不同参数化的相关问题。[124] 将带有非零挠率张量的爱因斯坦-嘉当引力应用到希格斯暴胀中。该模型存在大量参数，使得它与观测的对比内容丰富得多。另一方面，爱因斯坦-嘉当理论是引力理论中帕拉蒂尼方法最简单的自然推广，并且正如文献 [124] 所述，没有理由将它排除在引力基本理论的候选家族之外。

Density Matrix of the Universe and Initial Conditions for Inflation

宇宙密度矩阵与暴胀初始条件

As mentioned in the Introduction, the success of the Higgs inflation model does not extend to the solution of two big problems - explanation of initial conditions for inflation and necessity to rely on the more fundamental theory than the Standard Model of particles, which could be responsible for the quantum nature of gravity at the energy-scale characteristic of the quantum origin of inflationary Universe. The Standard Model can be regarded as an effective field theory representing a low-energy approximation to the theory of superstrings capable of probing the domain of Planckian scales, but suffering from the so-called landscape problem - enormous ambiguity in the choice of its vacua [54, 55] . At least naively, this problem might be regarded as a formulation of selection criteria that could restrict the energy domain of the physical realm of our nature including such a phenomenon as quantum origin of the Universe. There is a hope that such a criterion can come from quantum cosmology within the formalism which would impose bounds on this energy scale, and if we are lucky enough, this bound would be below the effective field theory cutoff so that we, in the absence of efficient nonperturbative methods, could proceed within the perturbation theory. Here we present the attempt to implement this idea in the form of the density matrix of the Universe [57-59, 129].

正如引言中所述，希格斯暴胀模型的成果并未涵盖两大问题的解决: 一是解释暴胀的初始条件，二是它必须依赖比粒子标准模型更基础的理论——在暴胀宇宙量子起源的特征能标下，引力的量子性质需要该基础理论来描述。标准模型可被视为超弦理论的低能近似有效场论，超弦理论能够探测普朗克尺度领域，但存在所谓的景观问题——真空选择存在极大的不确定性 [54, 55] 。至少从直观来看，该问题可以被看作是需要一套选择标准，来限制包含宇宙量子起源这一现象在内的大自然物理领域的能区。人们有望在量子宇宙学的形式体系中得到这套标准，该形式体系会对这个能标施加限制；如果足够幸运，这个限制会落在有效场论的截断之下，那么在缺乏有效非微扰方法的情况下，我们就可以继续在微扰论的框架内开展研究。本文我们尝试以宇宙密度矩阵的形式实现这一思路 [57-59, 129]。

Microcanonical Density Matrix of the Universe

宇宙的微正则密度矩阵

The first step in pursuit of the above program consists in the denial of a pure nature of the quantum state of the Universe and in its replacement by a density matrix. Among the reasons for this denial is that the choice of a certain quantum state, like the no-boundary or tunnelling one, is always associated with additional assumptions. So, in the spirit of Occam razor denying redundant assumptions, there is a room for being democratic and try handling all possible quantum states on equal footing, which in fact implies transition to their mixture - a density matrix. The very idea that instead of a pure quantum state, one can consider a density matrix in the form of the Euclidean quantum gravity path integral was pioneered by D. Page in [60]. However, Euclidean quantum gravity which, in particular, underlies the Hartle-Hawking no-boundary wave function can hardly be a fundamental concept. Rather, it should be derived as a calculational tool from canonical quantization in real physical spacetime [5], just like how it works in a conventional quantum field theory. Such a derivation might look as follows.

上述研究方案的第一步是否定宇宙量子态的纯态性质，将其替换为密度矩阵。之所以做出这种否定，原因之一在于，选择无边界态或隧穿态这类特定量子态始终伴随着额外假设。因此，本着奥卡姆剃刀剔除冗余假设的精神，我们完全可以采取更平等包容的思路，尝试将所有可能的量子态置于同等地位处理，这实际上就意味着我们需要过渡到这些态的混合态——密度矩阵。用密度矩阵替代纯量子态、将其表示为欧几里得量子引力路径积分形式的想法最早由 D. Page 在文献 [60] 中提出。然而，作为哈特-霍金无边界波函数基础的欧几里得量子引力很难称得上是一个基础概念，它更应该作为计算工具从真实物理时空的正则量子化中导出 [5]，就像传统量子场论中的情况一样。这一推导过程大致如下。

Consider instead of a one concrete solution $|\Psi\rangle$ of the Wheeler-DeWitt equation (1) a full set of them and construct the density matrix according to the following transition:

我们不考虑惠勒-德威特方程 (1) 的某一个特解 $|\Psi\rangle$，而是考虑它的全部解，并按照如下变换构造密度矩阵:

$$\Big|\Psi\rangle \to \hat{\rho} = \sum_{\text{all }|\Psi\rangle}\Big|\Psi\rangle\langle\Psi|, \tag{72}$$

where summation runs over all such solutions, $\hat{\mathcal{H}}|\Psi\rangle = 0$. Unfortunately, we do not yet know a precise operator realization of the set of the quantum Hamiltonian and momentum constraints in gravity theory $\hat{\mathcal{H}} = \left\{\hat{H}_\perp(\mathbf{x}), \hat{H}_a(\mathbf{x})\right\} \equiv \hat{H}_\mu$, which we will collectively denote by $\hat{H}_\mu$ with the condensed index $\mu = (\perp\mathbf{x}, a\mathbf{x})$ including the continuous spatial coordinate label. Therefore, it is hard to define the set of conditions functionally restricting this space of solutions, but one can formally proceed with the definition (72) treating it as a projector in terms of generalized operator-valued delta function,

其中求和遍历所有这类解 $\hat{\mathcal{H}}|\Psi\rangle = 0$。遗憾的是，我们目前尚未掌握引力理论 $\hat{\mathcal{H}} = \left\{\hat{H}_\perp(\mathbf{x}), \hat{H}_a(\mathbf{x})\right\} \equiv \hat{H}_\mu$ 中量子哈密顿约束和动量约束集合的精确算符实现，我们将这些约束统一记为 $\hat{H}_\mu$，其中凝聚指标 $\mu = (\perp\mathbf{x}, a\mathbf{x})$ 包含连续空间坐标标签。因此，我们很难给出对这个解空间加以函数限制的条件集合，但可以从形式上按定义 (72) 进行处理，将其视作广义算符值 δ 函数形式的投影算符，

$$\hat{\rho} = \frac{1}{Z}\delta\left(\hat{\mathcal{H}}\right), \; Z = \mathrm{tr}\,\delta\left(\hat{\mathcal{H}}\right), \tag{73}$$

$$\delta\left(\hat{\mathcal{H}}\right) = \prod_{\mu} \delta\left(\hat{H}_{\mu}\right)''. \tag{74}$$

Quotation marks indicate here that the product of non-commuting operators $\hat{H}_\mu$ should not be understood literally, but rather interpreted as an operatorial generalized function projecting onto the kernel of all the constraints that is satisfying the equation $\hat{H}_\mu \delta\left(\hat{\mathcal{H}}\right) = \delta\left(\hat{\mathcal{H}}\right)\hat{H}_\mu = 0$. Consistency of the definition of this projector follows from the involution algebra of these constraints - their commutator being their linear combination [129]. The commutator algebra of constraints does not form the Lie algebra of a group, so that it is impossible to define such a projector by integration over the group volume.

此处的引号表示，非对易算符 $\hat{H}_\mu$ 的乘积不应按字面理解，而应被解释为一个算符广义函数，它投影到所有约束的核空间，即满足方程 $\hat{H}_\mu \delta\left(\hat{\mathcal{H}}\right) = \delta\left(\hat{\mathcal{H}}\right)\hat{H}_\mu = 0$。该投影算符定义的自洽性来自这些约束的对合代数——它们的对易子本身就是自身的线性组合 [129]。约束的对易子代数不构成群的李代数，因此无法通过对群体积积分来定义这类投影算符。

However, the projector (74) can be constructed along the lines of the BFV quantization [127, 128] in the form of the path integral over BRST-extended set of fields [129]. Its matrix element in the functional coordinate representation of 3- metric coefficients and matter fields $q = (g_{ab}(\mathbf{x}), \phi(\mathbf{x}))$ turns out to be given by the canonical Faddeev-Popov path integral,

不过，投影算符 (74) 可以按照 BFV 量子化方案 [127, 128] 构造，形式为 BRST 延拓场集合上的路径积分 [129]。它在 3 度规系数和物质场 $q = (g_{ab}(\mathbf{x}), \phi(\mathbf{x}))$ 的泛函坐标表示下的矩阵元可由正则法捷耶夫-波波夫路径积分给出，

$$\left\langle q_+ \left|\delta\left(\hat{\mathcal{H}}\right)\right| q_- \right\rangle = \int_{q(t_\pm)=q_\pm} D[q,p,N] \exp\left[i\int_{t_-}^{t_+} dt\left(p\dot{q} - N^\mu H_\mu\right)\right], \tag{75}$$

with the canonical ADM form of the gravitational action [130] and the gauge-fixing procedure encoded in the integration measure $D[q,p,N]$. This is the integration over histories of gravitational and matter phase space variables (q,p) and Lagrange multipliers N^μ which are nothing but the ADM lapse and shift functions $N^\mu = \left(N^\perp(\mathbf{x}), N^a(\mathbf{x})\right)$. These histories interpolate between the arguments $q_\pm$ of the projector kernel [5].

其中引力作用量取正则 ADM 形式 [130]，规范固定过程被包含在积分测度 $D[q,p,N]$ 中。该积分是对引力和物质相空间变量 (q,p) 以及拉格朗日乘子 N^μ 的历史进行积分，拉格朗日乘子正是 ADM 移位和时移函数 $N^\mu = \left(N^\perp(\mathbf{x}), N^a(\mathbf{x})\right)$。这些历史连接投影算符核的两个宗量 $q_\pm$ [5]。

Note that the parameter t here, which looks exactly like a physical time variable in the canonical formalism of general relativity, originally arose as an operator-ordering parameter that allows one to account for non-Abelian nature of quantum constraints. Originally its role was to extend to the operator level with non-commuting $\hat{H}_\mu$, the c-number delta function $\int dN \exp\left(-iN^\mu H_\mu\right) = \prod_\mu \delta\left(H_\mu\right)$ in the representation of the Fourier integral over N^μ (single-time not the path-integral one).

请注意，此处参数 t 在广义相对论的正则形式中看起来完全是一个物理时间变量，它最初是作为算符排序参数引入的，用于处理量子约束的非阿贝尔性质。最初它的作用是将非对易 $\hat{H}_\mu$ 的 c 数 δ 函数 $\int dN \exp\left(-iN^\mu H_\mu\right) = \prod_\mu \delta\left(H_\mu\right)$ 拓展到算符层面，该 δ 函数出现在对 N^μ 的傅里叶积分表示中 (该积分是单时间积分，不是路径积分)。

A physical interpretation of the definition (73) is that it is the analogue of a microcanonical density matrix for the usual non-gauge dynamical system with a fixed value of the total conserved energy $E, \hat{\rho} = \delta\left(\hat{H} - E\right)/Z-$ equipartition of all physical states with one and the same energy value. In spatially closed cosmology there are no global conserved charges including its energy which is not being defined at all. Instead, the closed cosmology has local conserved objects - the constraints H_μ - whose only physically meaningful and mathematically consistent value is zero. Therefore, the density matrix (73) can be regarded as a definition of a microcanonical ensemble in quantum cosmology - a universal equipartition of all physical states satisfying the condition of local diffeomorphism invariance.

定义 (73) 的物理解释是，它对应常规非规范动力学系统的微正则密度矩阵，在这类系统中总守恒能量 $E, \hat{\rho} = \delta\left(\hat{H} - E\right)/Z-$ 取固定值，所有能量相同的物理态满足等概率分布。在空间闭合宇宙学中不存在整体守恒荷，宇宙的能量甚至完全没有定义。与之相对，闭合宇宙学存在局部守恒对象——约束 H_μ，它在物理上有意义且数学自洽的取值只有零。因此，密度矩阵 (73) 可以看作量子宇宙学中微正则系综的定义——对所有满足局部微分同胚不变性条件的物理态做统一等概率分布。

The partition function of this ensemble,

该系综的配分函数，

$$
\begin{aligned}
Z &= \int dq\mu(q) \left\langle q_+ \left| \delta\left(\hat{\mathcal{H}}\right) \right| q_- \right\rangle \Big|_{q_\pm = q} \\
&= \int_{\text{periodic}} D[q,p,N] \exp\left[i \int_{t_-}^{t_+} dt \left(p\dot{q} - N^\mu H_\mu \right) \right], \qquad (76)
\end{aligned}
$$

obviously implies path integration over closed periodic histories (the measure of the physical inner product $\mu(q)$ being absorbed in the path integration measure $D[q,p,N]$ of such histories [129]). Note that despite a usually exploited relation between microcanonical thermodynamics and imaginary time formalism, the path integral here is still over fields in Lorentzian signature spacetime.

显然等价于对闭合周期历史做路径积分，其中物理内积的测度 $\mu(q)$ 被吸收进这类历史的路径积分测度 $D[q,p,N]$ 中 [129]。请注意，尽管微正则热力学与虚时间形式之间存在人们常用的关联，但此处的路径积分仍然是对洛伦兹号差时空下的场做积分。

The further transformation of the path integral consists in Gaussian integration over momenta and the use of relativistic gauge conditions. This eventually allows one to assemble the initially $(3+1)$ -splitted fields $q = (g_{ab}(t,\mathbf{x}), \phi(t,\mathbf{x}))$ and $N^\mu = (N(t,\mathbf{x}), N^a(t,\mathbf{x}))$ into covariant multiplets of spacetime metric $g_{\mu\nu}(x)$ and matter $\phi(x)$ variables and rewrite the partition function as a covariant Lagrangian path integral over periodic fields on the manifold with a variable $x^0 = t$ compactified to a circle S^1 ,

路径积分的进一步变换包括对动量做高斯积分，以及使用相对论规范条件。这最终可以将最初被 $(3+1)$ 拆分的场 $q=(g_{ab}(t,\mathbf{x}),\phi(t,\mathbf{x}))$ 和 $N^{\mu}=\ (N(t,\mathbf{x}),N^{a}(t,\mathbf{x}))$ 组装为时空度规 $g_{\mu\nu}(x)$ 和物质 $\phi(x)$ 变量的协变多重态，并将配分函数改写为协变拉格朗日路径积分，积分对象是拓扑为可变维数 $x^0=t$ 紧致化为圆周 S^1 的流形上的周期场，

$$Z=\int_{\text{periodic}} D\left[g_{\mu\nu},\phi\right]e^{iS[g_{\mu\nu},\phi]}. \tag{77}$$

The notion of imaginary time or Euclidean spacetime arises only at the level of semiclassical expansion. This is a usually accepted assumption that the integrand of a path integral has two types of analyticity - one with respect to the extension of integration fields to their complex values and another one is with respect to transition of their spacetime arguments into the complex plane of time (In gravity theory the second type of analyticity can be associated with the first type when the Euclidean spacetime is attained not at imaginary values of time parameter, but at the imaginary value of the lapse function.). Therefore, if the stationary point of the path integral (77) is not available, that is, no periodic solutions of equations of motion for $S\left[g_{\mu\nu},\phi\right]$ exist in Lorentzian signature spacetime, then such periodic solutions can be looked for in imaginary time. And if they exist, they can serve as saddle point configurations in Euclidean spacetime with the negative of the Euclidean action $-S_{\text{E}}=iS$ in the exponential of (77).

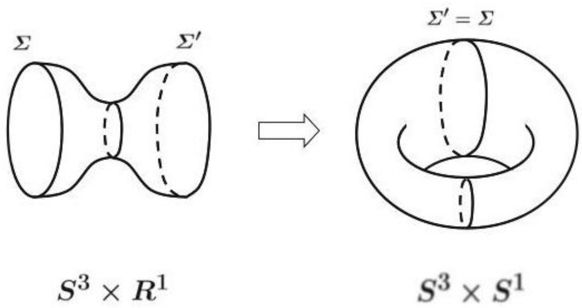
虚时间或欧几里得时空的概念仅在半经典展开层面才会出现。人们普遍接受的假设是，路径积分的被积函数具有两类解析性: 一类是针对将积分场延拓到复数值的解析性，另一类是针对将时空参数过渡到复时间平面的解析性 (在引力论中，当欧几里得时空不是通过时间参数取虚值得到，而是通过移函数取虚值得到时，第二类解析性可以和第一类关联起来)。因此，如果路径积分 (77) 不存在驻点，即洛伦兹号差时空中不存在 $S\left[g_{\mu\nu},\phi\right]$ 运动方程的周期解，就可以在虚时间中寻找这类周期解。如果这类解存在，它们就可以作为欧几里得时空里的鞍点构型，对应 (77) 的指数项中取欧几里得作用量 $-S_{\text{E}}=iS$ 的负值。

Such a solution in Euclidean spacetime, or cosmological instanton, for spatially closed cosmology with S^3 topology of spatial sections will have the 4-dimensional topology $S^3\times S^1$. Its origin from the corresponding instanton for the density matrix with the topology $S^3\times R^1$ is graphically represented on Fig. 4 as a result of gluing two spatial hypersurfaces $\sum$ and $\sum'$ associated with two arguments of the density matrix (73). At the glued minimal surfaces $\sum$ and $\sum'$, labelled by the values τ and τ' , respectively, of Euclidean time, all fields have vanishing time derivatives, which follows from matching periodicity conditions. Therefore, a real valued instanton solution can be analytically continued to the imaginary axes of the Euclidean time (or real axes of the Lorentzian time t), $\tau\to\tau+it$ and $\tau'\to\tau'+it'$, and the resulting solutions as functions of t and t' will also be real valued. The combined Euclidean-Lorentzian instanton is then graphically represented on Fig. 5 where its Lorentzian part is depicted by dashed lines.

对于空间截面拓扑为 S^3 的空间闭合宇宙学，这类欧氏时空解即宇宙瞬子，将具有 4 维拓扑 $S^3\times S^1$ 。它源自对应密度矩阵的拓扑为 $S^3\times R^1$ 的瞬子，如图 4 所示，是将与密度矩阵 (73) 的两个自变量关联的两个空间超曲面 $\sum$ 和 $\sum'$ 粘合得到的。在分别由欧氏时间值 τ 和 τ' 标记的粘合极小曲面 $\sum$ 和 $\sum'$ 上，所有场的时间导数都为零，这由匹配周期性条件推导得出。因此，实值瞬子解可以解析延拓到欧氏时间的虚轴 (或洛伦兹时间 t 的实轴) $\tau\to\tau+it$ 和 $\tau'\to\tau'+it'$ ，得到的作为 t 和 t' 函数的解也仍是实值的。这种欧氏-洛伦兹混合瞬子如图 5 所示，其中洛伦兹部分用虚线绘制。

The last picture demonstrates the difference between the pure no-boundary state from the density matrix one. The no-boundary state, which originally is defined by the Euclidean quantum gravity path integral, can also be represented by the density matrix having a factorizable form and corresponding to the instanton shown on Fig. 6. What distinguishes it from the impure state described by the density matrix is that its underlying instanton is the union of two disjoint manifolds. The instanton bridge between the surfaces $\sum$ and $\sum$' of Fig. 5 is broken, which implies absence of correlations between the quantum states of fields on these surfaces.

最后这张图说明了无边界纯态与密度矩阵描述的态的区别。最初由欧氏量子引力路径积分定义的无边界态，也可以用可因式化形式的密度矩阵表示，对应图 6 所示的瞬子。它和密度矩阵描述的非纯态的区别在于，其底层瞬子是两个不相交流形的并集。图 5 中曲面 $\sum$ 和 $\sum$' 之间的瞬子桥断裂，意味着这两个曲面上场的量子态之间不存在关联。



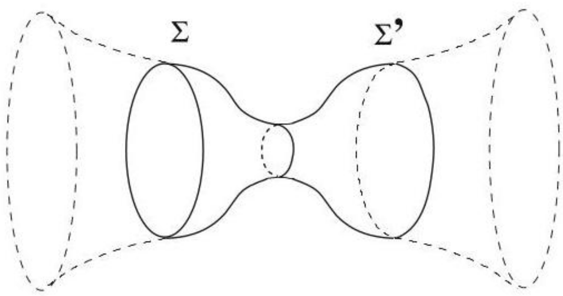


Fig. 4 Origin of the partition function instanton from the density matrix instanton by the procedure of gluing the boundaries $\sum$ and $\sum'$ – tracing the density matrix

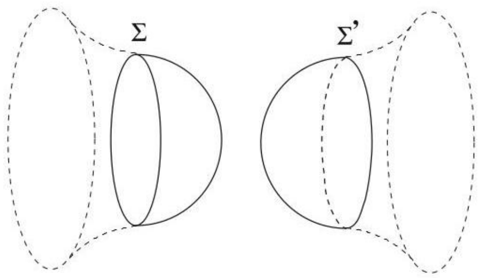
Fig. 4 通过粘合边界 $\sum$ 和 $\sum'$ – 求密度矩阵迹，从密度矩阵瞬子得到配分函数瞬子的过程

Fig. 5 Picture of instanton representing the density matrix. Dashed lines depict the Lorentzian Universe nucleating from the instanton at the minimal surfaces $\sum$ and $\sum'$

Fig. 5 表示密度矩阵的瞬子示意图。虚线描绘了在极小曲面 $\sum$ 和 $\sum'$ 处从瞬子成核产生的洛伦兹宇宙

Fig. 6 Density matrix of the pure Hartle-Hawking state represented by the union of two no-boundary vacuum instantons

Fig. 6 由两个无边界真空瞬子的并集表示的哈特-霍金纯态密度矩阵



Thus, the starting point of the above construction [57-59] is the density matrix $\hat{\rho}$ with two surfaces carrying its field arguments. These surfaces semiclassically are the boundaries of either Euclidean or Lorentzian spacetime, depending on the relevant size of the scale factor. The entire saddle point solution for $\hat{\rho}$ consists of the Euclidean spacetime interpolating between them or of the Euclidean spacetime between $\sum$ and $\sum'$, sandwiched between the two layers of the Lorentzian spacetime, respectively. These two layers interpolate from $\sum$ to the unprimed argument of the density matrix and from $\sum'$ to its primed argument and correspond in the density matrix to the chronological and anti-chronological evolution factors of the wellknown Schwinger-Keldysh technique for expectation values [125, 126]. When calculating the statistical sum trace, these two factors cancel out in view of unitarity, and the only contribution that remains comes from the Euclidean domain between the Euclidean-Lorentzian transition surfaces $\sum$ and $\sum'$. These surfaces are uniquely determined from the condition of smooth periodicity in the Euclidean time on the compact S^1, or as two turning points of the Euclidean trajectory for $a(\tau)$.

因此，上述构造 [57-59] 的出发点是带有两个承载场自变量曲面的密度矩阵 $\hat{\rho}$ 。根据标度因子的对应大小，半经典上这些曲面是欧氏时空或洛伦兹时空的边界。$\hat{\rho}$ 的整个鞍点解，对应两种情况: 要么是介于两个曲面之间的欧氏时空，要么是介于 $\sum$ 和 $\sum'$ 之间的欧氏时空夹在两层洛伦兹时空之间。这两层洛伦兹时空分别从 $\sum$ 演化到密度矩阵的非带撇自变量，从 $\sum'$ 演化到密度矩阵的带撇自变量，在密度矩阵中对应著名施温格-凯尔迪什期望值方法中的时序和反时序演化因子 [125, 126]。计算统计和迹时，由于幺正性这两个因子相互抵消，仅剩来自欧氏-洛伦兹过渡曲面 $\sum$ 和 $\sum'$ 之间欧氏区域的贡献。这些曲面可由紧致 S^1 上欧氏时间的光滑周期性条件唯一确定，也可作为 $a(\tau)$ 欧氏轨迹的两个转折点唯一确定。

Effective Action and Cosmological Bootstrap in CFT-Driven Cosmology

共形场论驱动宇宙学中的有效作用量与宇宙学自举

Productive application of the above construction can be demonstrated in the model of inflation driven by the conformal field theory. This is the Einstein gravity model with a primordial cosmological constant Λ and the matter sector dominated by a large number of fields Φ conformally coupled to gravity - conformal field theory (CFT) with the action $S_{CFT}[g_{\mu\nu}, \Phi]$,

上述构造的有效应用可以在共形场论驱动的暴胀模型中得到展示。这是一个存在原初宇宙学常数 Λ ，且物质部分由大量共形耦合引力的场 Φ ——即作用量为 $S_{CFT}[g_{\mu\nu}, \Phi]$ 的共形场论 (CFT)——主导的爱因斯坦引力模型,

$$S_{\mathrm{E}}\left[g_{\mu\nu},\Phi\right]=-\frac{M_P^2}{2}\int d^4xg^{1/2}\left(R-2\Lambda\right)+S_{CFT}\left[g_{\mu\nu},\Phi\right]. \tag{78}$$

An important property which allows one to overstep the limits of the usual semiclassical expansion consists here in the possibility to omit integration over conformally non-invariant matter fields and spatially inhomogeneous metric modes on top of a dominant contribution of these conformal species. Integrating them out, one obtains the effective gravitational action $S_{\mathrm{eff}}\left[g_{\mu\nu}\right]$ which differs from (78) by $S_{CFT}\left[g_{\mu\nu},\Phi\right]$ replaced with $\Gamma_{CFT}\left[g_{\mu\nu}\right]$ - the effective action of Φ on the background of $g_{\mu\nu}$,

> 这里存在一个能让我们突破常规半经典展开限制的重要性质: 由于这些共形场类的贡献占主导，我们可以无需对共形非不变物质场和空间非均匀度规模式进行积分。将这些自由度积分掉后，我们就能得到有效引力作用量 $S_{\mathrm{eff}}\left[g_{\mu\nu}\right]$，它和式 (78) 的区别在于 $S_{CFT}\left[g_{\mu\nu},\Phi\right]$ 被替换为 $\Gamma_{CFT}\left[g_{\mu\nu}\right]$，即 Φ 在 $g_{\mu\nu}$ 背景下的有效作用量,

$$e^{-\Gamma_{CFT}\left[g_{\mu\nu}\right]}=\int D\Phi e^{-S_{CFT}\left[g_{\mu\nu},\Phi\right]}. \tag{79}$$

On Friedmann-Robertson-Walker (FRW) background, this action is exactly calculable by using the local conformal transformation to the static Einstein universe and well-known local trace anomaly. The resulting $\Gamma_{CFT}\left[g_{\mu\nu}\right]$ turns out to be the sum of the anomaly contribution and free energy of conformal matter fields at the effective temperature determined by the circumference of the compactified time dimension S^1 . This is an important calculational advantage provided by the local Weyl invariance of quantum matter.

> 在弗里德曼-罗伯逊-沃克 (FRW) 背景下，我们可以通过局域共形变换转换到静态爱因斯坦宇宙，再结合众所周知的局域迹反常对该作用量精确计算。最终得到的 $\Gamma_{CFT}\left[g_{\mu\nu}\right]$ 是反常贡献与共形物质场在有效温度下自由能的总和，该有效温度由紧化时间维度 S^1 的周长确定。这就是量子物质的局域外尔不变性带来的重要计算优势。

Physics of the CFT-driven cosmology is entirely determined by this effective action. Solutions of its equations of motion, which give a dominant contribution to the statistical sum, are the cosmological instantons of $S^1\times S^3$ topology, which have the Friedmann-Robertson-Walker metric

> 共形场论驱动宇宙学的物理完全由该有效作用量决定。对配分求和给出主导贡献的运动方程解，是拓扑为 $S^1\times S^3$ 的宇宙学瞬子，其度规为弗里德曼-罗伯逊-沃克度规

$$ds^2=N^2\left(\tau\right)d\tau^2+a^2\left(\tau\right)d^2\Omega^{(3)} \tag{80}$$

with a periodic lapse function $N\left(\tau\right)$ and scale factor $a\left(\tau\right)$ - functions of the Euclidean time belonging to the circle S^1 [57]. These instantons can be interpreted as initial conditions for the cosmological evolution $a_L\left(t\right)$ in the physical Lorentzian spacetime. This evolution follows from $a\left(\tau\right)$ by analytic continuation to real time $t, a_L\left(t\right)=a\left(\tau_*+it\right)$, at the point of the maximum value of the Euclidean scale factor $a_+=a\left(\tau_*\right)$. Looking ahead to the above formulated goals of this setup, let us say that these instantons will exist only in the finite range of Λ . Under the assumption that Λ is a function of the inflaton field staying, say, in slow roll regime and exiting it according to conventional inflation scenario, this restriction would mean the formation of initial conditions of inflation. On the other hand, this restriction may be interpreted as the selection criterion in the landscape of stringy vacua.

具有周期性流逝函数 $N(\tau)$ 和标度因子 $a(\tau)$ ——二者属于圆 S^1 上的欧几里得时间函数 [57]。这些瞬子可以解释为物理洛伦兹时空中宇宙演化 $a_L(t)$ 的初始条件。该演化是在欧几里得标度因子 $a_+ = a(\tau_*)$ 的最大值点，将 $a(\tau)$ 解析延拓到实时间 $t, a_L(t) = a(\tau_* + it)$ 得到的。展望该框架的既定目标，我们可以说这些瞬子仅存在于 Λ 的有限范围内。假设 Λ 是暴胀子场的函数，且该场处于慢滚 regime 并按照传统暴胀场景退出慢滚，那么这个限制就意味着暴胀初始条件的形成。另一方面，该限制也可以被解释为弦真空景观中的选择标准。

The realization of this program is as follows. For cosmology with the metric (80), its effective action reads [57]

该方案的实现如下。对于度规为 (80) 的宇宙学，其有效作用量可写为 [57]

$$S_{\text{eff}}[a,N] = 6\pi^2 M_P^2 \int_{S^1} d\tau N \Big\{ -aa'^2 - a + \frac{\Lambda}{3}a^3$$

$$+B\left(\frac{a'^2}{a} - \frac{a'^4}{6a}\right) + \frac{B}{2a}\Big\} + F(\eta), \tag{81}$$

$$F(\eta) = \sum_{\omega} (\pm 1) \ln\left(1 \mp e^{-\omega\eta}\right), \ \eta = \int_{S^1} \frac{d\tau N}{a}, \tag{82}$$

where $a' \equiv da/Nd\tau$. The first three terms in curly brackets of (81) represent the Einstein action with a fundamental cosmological constant $\Lambda \equiv 3H^2$ (H is the corresponding Hubble parameter). The constant B is a coefficient of the contributions of the conformal anomaly and vacuum (Casimir) energy ($B/2a$) on a conformally related static Einstein spacetime mentioned in the Introduction. This constant,

其中 $a' \equiv da/Nd\tau$ 。式 (81) 花括号中的前三项代表带有基础宇宙学常数 $\Lambda \equiv 3H^2$ 的爱因斯坦作用量 (H 是对应的哈勃参数)。常数 B 是引言中提到的、共形相关静态爱因斯坦时空上共形反常与真空 (卡西米尔) 能量贡献 ($B/2a$) 的系数。该常数

$$B = \frac{\beta}{8\pi^2 M_P^2}, \tag{83}$$

expresses via the coefficient β of the Gauss-Bonnet term $E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2$ in the trace anomaly of conformal matter fields

可以通过共形物质场迹反常中高斯-博内项 $E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2$ 的系数 β 表示

$$g_{\mu\upsilon}\frac{\delta\Gamma_{\text{CFT}}}{\delta g_{\mu\upsilon}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\nabla^2 R + \beta E + \gamma C^2_{\mu\upsilon\alpha\beta}\right). \tag{84}$$

The effective action is independent of the anomaly coefficients α and γ , because Weyl tensor $C_{\mu\upsilon\alpha\beta}$ identically vanishes for any Friedmann metric and it is assumed that α can be renormalized to zero by a local counterterm $\sim \int d^4x g^{1/2} R^2$. This, in particular, guarantees absence of higher-derivative terms in (81) - non-ghost nature of the scale factor - and simultaneously endows the renormalized Casimir energy with a special value proportional to $B/2 = \beta/16\pi^2 M_P^2$ [57]. Both of these properties are critically important for the instanton solutions of effective equations.

有效作用量与反常系数 α 和 γ 无关，因为魏尔张量 $C_{\mu\nu\alpha\beta}$ 对任何弗里德曼度量都恒等于零，且我们认为 α 可通过局部抵消项 $\sim \int d^4xg^{1/2}R^2$ 重整化为零。这尤其保证了式 (81) 中不存在高阶导数项——标度因子无鬼，同时使重整化的卡西米尔能量获得一个与 $B/2 = \beta/16\pi^2M_P^2$ 成正比的特殊值 [57]。这两个性质对有效方程的瞬子解都至关重要。

Finally, $F(\eta)$ in (82) is the free energy of conformal fields on static Einstein universe space (to which the conformal rescaling mentioned above was done) - a typical boson or fermion sum over CFT field oscillators with energies ω on a unit 3-sphere, η playing the role of the inverse temperature - an overall circumference of S^1 in the $S^1 \times S^3$ instanton geometry, which is calculated in units of the conformal time (82).

最后，式 (82) 中的 $F(\eta)$ 是静态爱因斯坦宇宙空间 (即上文做共形标度变换后的空间) 上共形场的自由能，是单位三维球面上能量为 ω 的共形场论场振子的典型玻色子或费米子求和，其中 η 充当逆温度，即 $S^1 \times S^3$ 瞬子几何中 S^1 的总周长，以共形时间 (82) 为单位计算。

The statistical sum (77) is dominated by the solutions of the effective equation, $\delta S_{\text{eff}}/\delta N(\tau) = 0$, which in the cosmic time gauge ($N = 1$) reads

统计求和 (77) 由有效方程的解 $\delta S_{\text{eff}}/\delta N(\tau) = 0$ 主导，该方程在宇宙时间规范 ($N = 1$) 中形式为

$$-\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} - B\left(\frac{\dot{a}^4}{2a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{\Lambda}{3} + \frac{C}{a^4}, \dot{a} = \frac{da}{d\tau}, \tag{85}$$

$$C = \frac{B}{2} + \frac{1}{6\pi^2 M_P^2}\sum_\omega \frac{\omega}{e^{\omega\eta} \mp 1}. \tag{86}$$

This is the modification of the Euclidean Friedmann equation by the conformal anomaly term $\sim B$, the radiation energy term C/a^4 which is the sum of the Casimir energy $\sim B/2$ and the energy of thermally excited particles with the inverse temperature η given by (82). Note that the constant C is a nonlocal functional of the history $a(\tau)$ - Eq. (86) plays the role of the bootstrap equation for the amount of radiation determined by the background on top of which this radiation evolves and produces a back reaction.

这是欧几里得弗里德曼方程经共形反常项 $\sim B$ 、辐射能量项 C/a^4 修改后的形式，其中辐射能量项是卡西米尔能量 $\sim B/2$ 与逆温度为 η (由式 (82) 给出) 的热激发粒子能量之和。注意常数 C 是演化历史 $a(\tau)$ 的非局域泛函——式 (86) 充当自举方程，确定了辐射的总量: 辐射在背景上演化并产生反作用，其总量由该背景决定。

Equation (85) can be solved for $\dot{a}^2$,

式 (85) 可对 $\dot{a}^2$ 求解,

$$\dot{a}^2 = \sqrt{\frac{\left(a^2 - B\right)^2}{B^2} + \frac{2H^2}{B}\left(a_+^2 - a^2\right)\left(a^2 - a_-^2\right)} - \frac{a^2 - B}{B}\ , \tag{87}$$

$$a_\pm^2 \equiv \frac{1 \pm \sqrt{1 - 4CH^2}}{2H^2}, \tag{88}$$

to give a periodic oscillation of a between its maximal and minimal values $a_\pm$, provided that at a_- we have a turning point with a vanishing $\dot{a}$, which means that $a_-^2 > B$. This inequality together with the requirement of reality of turning points $a_\pm$ immediately yields the following restrictions on the range of H^2 and C :

得到 a 在最大值 $a_\pm$ 和最小值之间的周期性振荡，前提是在 a_- 处存在一个 $\dot{a}$ 为零的转向点，这满足条件 $a_-^2 > B$。该不等式加上转向点 $a_\pm$ 为实数的要求，立即对 H^2 和 C 的取值范围给出如下限制:

$$H^2 \leq \frac{1}{2B}, \ \frac{1}{4H^2} \geq C \geq B - B^2H^2. \tag{89}$$

Garland Instantons and Elimination of Infrared Catastrophe

花环瞬子与红外灾难的消除

The solutions of this integro-differential equation give rise to the set of periodic $S^3 \times S^1$ instantons with the oscillating scale factor - garlands - that can be regarded as the thermal alternative to the Hartle-Hawking instantons [57,59,62]. The scale factor oscillates m times ($m = 1, 2, 3, \ldots$) between the maximum and minimum values (88), $a_- \leq a(\tau) \leq a_+$, so that the full period of the conformal time (82) is the $2m$ -multiple of the integral between the two neighbouring turning points of $a(\tau)$, $\dot{a}(\tau_\pm) = 0$,

该积分微分方程的解给出了一组具有振荡标度因子的周期性 $S^3 \times S^1$ 瞬子——花环——可视为哈特尔-霍金瞬子的热学替代方案 [57,59,62]。标度因子在最大值和最小值 (88) 之间振荡 m 次 ($m = 1, 2, 3, \ldots$)，$a_- \leq a(\tau) \leq a_+$，因此共形时间 (82) 的完整周期是 $a(\tau), \dot{a}(\tau_\pm) = 0$, 两个相邻转折点之间积分的 $2m$ 倍

$$\eta = 2m \int_{a_-}^{a_+} \frac{da}{\dot{a}a}. \tag{90}$$

This value of η is finite and determines effective temperature $T = 1/\eta$ as a function of $G = 1/8\pi M_P^2$ and $\Lambda = 3H^2$. This is the artifact of a microcanonical ensemble in cosmology with only two freely specifiable dimensional parameters - the gravitational and cosmological constants (Fig. 7).

η 的该取值是有限的，它确定了有效温度 $T = 1/\eta$ 是 $G = 1/8\pi M_P^2$ 和 $\Lambda = 3H^2$ 的函数。这是仅含两个可自由设定维度参数——引力常数与宇宙学常数——的宇宙学中微正则系综的人为产物 (图 7)。

According to the bounds (89), these garland-type instantons exist only in the limited range of the cosmological constant $\Lambda = 3H^2$ [57]. They belong to the domain in the two-dimensional plane of the Hubble constant H^2 and the amount of radiation constant C . In this domain they form a countable, $m = 0, 1, 2, \ldots$, sequence of one-parameter families - curves interpolating between the lower straight line boundary $C = B - B^2H^2$ and the upper hyperbolic boundary $C = 1/4\mathrm{H}^2$. Each curve corresponds to respective m -folded instantons of the above type. Therefore, the range of admissible values of Λ ,

根据界 (89)，这类花环型瞬子仅存在于宇宙学常数 $\Lambda = 3H^2$ 的有限范围内 [57]。它们属于哈勃常数 H^2 与辐射常数 C 构成的二维平面中的一个区域。在此区域内，它们构成可数的 $m = 0, 1, 2, \ldots$ 单参数族序列——曲线插值于下直线边界 $C = B - B^2H^2$ 和上双曲线边界 $C =$ 1/4H^2 之间。每条曲线对应上述类型的对应 m 折叠瞬子。因此，Λ 容许值的范围，

$$\Lambda_{\min} \le \Lambda \le \Lambda_{\max} = \frac{12\pi^2 M_P^2}{\beta} = \frac{3}{2B}, \tag{91}$$

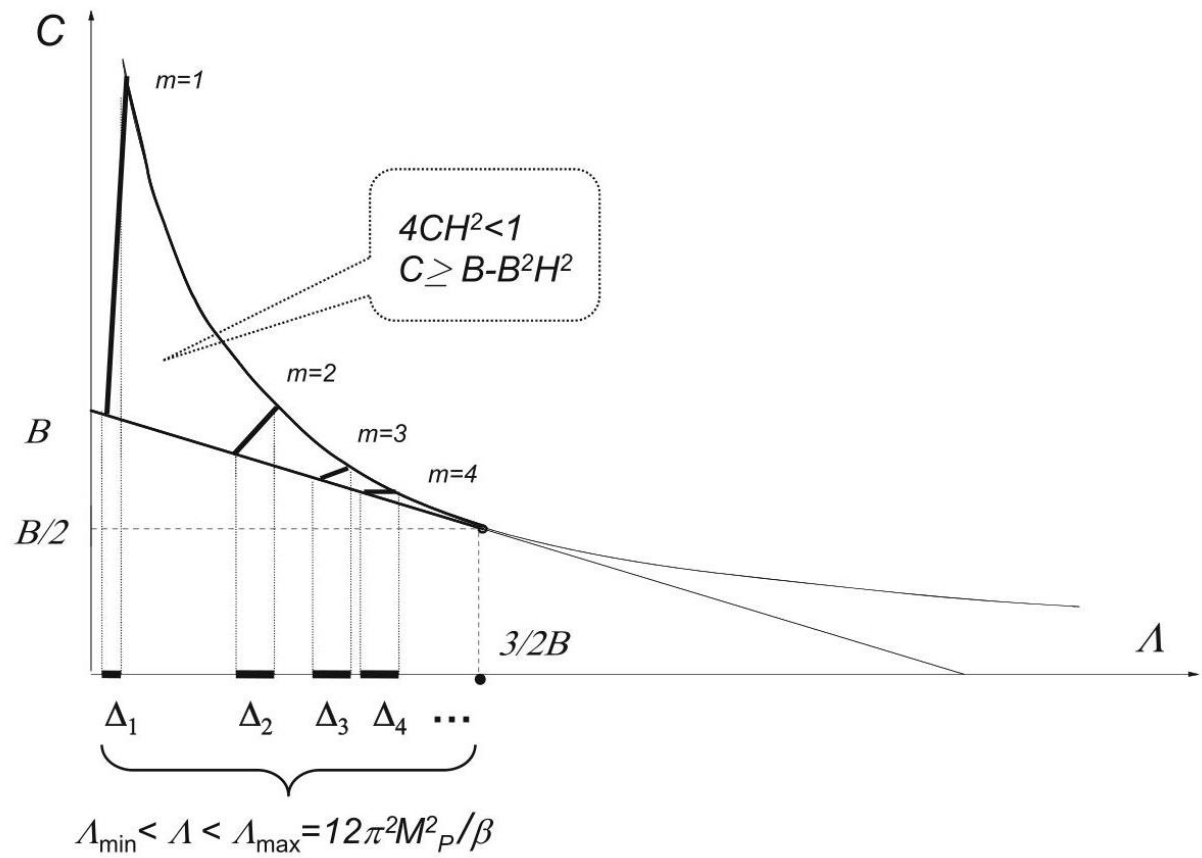


Fig. 7 Band range of garland instantons formed by projections Δ_m of their m -folded one-parameter families onto the axis of $\Lambda = 3H^2$

图 7 由 Δ_m 将其 m 折叠单参数族投影到 $\Lambda = 3H^2$ 轴形成的花环瞬子带状范围

has a band structure, each band Δ_m being a projection of the m th curve to the Λ axis. The sequence of bands of ever narrowing widths with $m \to \infty$ accumulates at the upper bound of this range $H^2_{\max} = 1/2B$. The lower bound $H^2_{\min}-$ the lowest point of the $m = 1$ family - can be obtained numerically for any field content of the model.

具有能带结构, 每个能带 Δ_m 都是第 m 条曲线在 Λ 轴上的投影。宽度逐渐变窄的能带序列随 $m \to \infty$ 累积在该范围的上界 $H^2_{\max} = 1/2B$ 处。下界 $H^2_{\min}-$ ——即 $m = 1$ 族的最低点——可通过数值方法对模型的任意场内容求得。

Another set of solutions follows from rewriting the effective equation (85) in the form (retaining in contrast to (87) both signs of the square root, which is possible in the other range of $a^2, a^2 < B$)

另一组解可通过将有效方程 (85) 重写为如下形式得到 (与 (87) 不同，保留平方根的两种符号，这在 $a^2, a^2 < B$ 的另一范围内是可行的)

$$\dot{a}^2 = 1 - \frac{a^2}{B}\left(1 \pm \sqrt{1 - 2BH^2 - \frac{B(2C-B)}{a^4}}\right) \tag{92}$$

and putting $C = B/2$ - absence of radiation, cf. Eq. (86),(without radiation in Lorentzian signature spacetime, this solution was derived in [131, 132]). This equation then reduces to [133]

并取 $C = B/2$ ——即无辐射，参见式 (86)，(洛伦兹号差时空中无辐射情况下该解已在 [131,132] 中导出)。此时方程约化为 [133]

$$\dot{a}^2 = 1 - H_\pm^2 a^2,\ H_\pm^2 = \frac{1 \pm \sqrt{1 - 2BH^2}}{B} = \frac{1}{a_\mp^2}\bigg|_{C=B/2}. \tag{93}$$

Obviously, the solutions to these two equations, $a(\tau) = \sin(H_\pm \tau)/H_\pm$, represent spherical Euclidean instantons $S^4_\pm$ of the radii $a_\mp$, respectively, or the strings of such spheres touching each other at their poles and forming a 'necklace' with any number of such spherical beads [133]. Note that the value of $C = B/2$ is consistent with the bootstrap equation (86), because the time period for such a necklace consisting of m beads,

显然，这两个方程的解 $a(\tau) = \sin(H_\pm \tau)/H_\pm$ 分别对应半径为 $a_\mp$ 的球形欧几里得瞬子 $S^4_\pm$ ，或是由多个这类球体在极点处彼此接触形成的串，构成含任意多个球形“珠子”的“项链”[133]。注意 $C = B/2$ 的取值自举方程 (86) 一致，因为这种由 m 个珠子组成的项链的时间周期，

$$\eta = 2m\int_0^{a_\pm} \frac{da}{\dot{a}a} = \infty, \tag{94}$$

diverges at the poles of spherical beads, where they touch each other - the range of integration over a in contrast to Eq. (90) is a multiple of the range between $a = 0$ at the pole of the 4-sphere $S^4_\pm$ and its value $a_\mp$ at the equator of $S^4_\pm$. Therefore, both $F(\eta)$ and $dF(\eta)/d\eta$ vanish and give in view of the bootstrap equation (86) the value of $C = B/2$.

在球形珠的极点处发散，球形珠在此处相互接触——与式 (90) 不同，对 a 的积分范围是 4 球面 $S^4_\pm$ 极点处 $a = 0$ 与其赤道处 $a_\mp$ 之间范围的倍数。因此，$F(\eta)$ 和 $dF(\eta)/d\eta$ 均为零，结合自举方程 (86) 得到 $C = B/2$ 的值。

These vacuum (or zero temperature, $1/\eta = 0$) necklace instantons existing for all values of $\Lambda = 3H^2 > 0$ are, however, not interesting because their contribution to the statistical sum is suppressed to zero by their infinite positive Euclidean action. For $B > 0$ the on-shell value of the action (81),

这些对所有 $\Lambda = 3H^2 > 0$ 取值都存在的真空 (或零温，$1/\eta = 0$) 项链瞬子，其实并没有研究意义，因为它们对统计和的贡献被无限正欧几里得作用压制为零。对于 $B > 0$ ， 作用量 (81) 的在壳值为

$$\Gamma_0 = F(\eta) - \eta F'(\eta) + 4m_P^2 \int_{S^1} \frac{d\tau}{a} \dot{a}^2 \left(B - a^2 - \frac{B\dot{a}^2}{3} \right) \to +\infty, \tag{95}$$

diverges to $+\infty$ at the poles of necklace beads with $a = 0$, where $|\dot{a}| = 1$ and $B - B\dot{a}^2/3 > 0$. Thus, the CFT cosmology scenario is free from infrared catastrophe of vacuum no-boundary instantons, which would otherwise have a negative tree-level Euclidean action (proportional to $-1/\Lambda \to -\infty$ at $\Lambda \to 0$) and which would imply that the origin of an infinitely big Universe is infinitely more probable than that of a finite one. Elimination of this infrared catastrophe is the quantum effect of the trace anomaly which flips the sign of the effective action and sends it to $+\infty$ [57, 65, 133].

在带有 $a = 0$ 的项链珠极点处发散至 $+\infty$ ，其中 $|\dot{a}| = 1$ 和 $B - B\dot{a}^2/3 > 0$ 。因此，CFT 宇宙学场景不存在真空无边界瞬子的红外灾难——若不存在上述效应，这类瞬子会拥有负的树级欧几里得作用量 (正比于 $\Lambda \to 0$ 处的 $-1/\Lambda \to -\infty$)，还会推导出无限大宇宙的起源概率比有限宇宙高得多的结论。这种红外灾难的消除是迹反常的量子效应，迹反常翻转了有效作用量的符号，将其推向 $+\infty$ [57, 65, 133]。

Inflation and the Hierarchy of Planck and Inflation Scales

暴胀与普朗克能标和暴胀能标的层级

The inflation stage in this model starts after the 'nucleation' of the system from the gravitational instanton at the $2m$ -th turning point of the m -folded garland, $\tau_* = 2m\int_{a_-}^{a_+} da/\dot{a}$ (cf. Eq. (90)). The Lorentzian time history of the scale factor $a_L(t)$ originates by the analytic continuation of the Euclidean solution $a(\tau)$ to $a_L(t) = a(\tau_* + it)$. This leads to the replacement of oscillatory behaviour of $a(\tau)$ by quasi-exponentially growing $a_L(t)$. When solved with respect to $\dot{a}^2$, the equation (85) can be converted to the form somewhat different from (87). In the Lorentzian domain with $a(\tau_* + it) = a_L(t)$ and $\dot{a}^2(\tau_* + it) = -\dot{a}_L^2(t)$, this form literally reads as general relativistic Friedmann equation:

该模型的暴胀阶段始于引力瞬子对系统的“成核”之后，发生在 $2m$ 折花环的 m 个转折点，$\tau_* = 2m\int_{a_-}^{a_+} da/\dot{a}$ (参见式 (90))。标度因子 $a_L(t)$ 的洛伦兹时间演化史源自对欧几里得解 $a(\tau)$ 向 $a_L(t) = a(\tau_* + it)$ 的解析延拓。这使得 $a(\tau)$ 的振荡行为被准指数增长的 $a_L(t)$ 取代。当以 $\dot{a}^2$ 为求解变量时，方程 (85) 可以转化为与 (87) 略有不同的形式。在满足 $a(\tau_* + it) = a_L(t)$ 和 $\dot{a}^2(\tau_* + it) = -\dot{a}_L^2(t)$ 的洛伦兹区域中，该形式正是广义相对论的弗里德曼方程:

$$\frac{\dot{a}_L^2}{a_L^2} + \frac{1}{a_L^2} = \frac{\varepsilon}{3M_{\rm eff}^2(\varepsilon)}, \ \varepsilon = M_P^2\Lambda + \frac{1}{2\pi^2 a_L^4} \sum_\omega \frac{\omega}{e^{\omega\eta} \mp 1}, \tag{96}$$

$$M_{\rm eff}^2(\varepsilon) = \frac{M_P^2}{2} \left(1 + \sqrt{1 - \frac{\beta\varepsilon}{12\pi^2 M_P^4}} \right), \tag{97}$$

with the effective Planck mass $M_{\rm eff}(\varepsilon)$ depending on the full matter density ε . This matter density includes together with the contribution of the cosmological constant $\Lambda = 3H^2$ also the primordial radiation of the conformal cosmology (but does not include Casimir energy totally screened due to the degravitating effect of conformal anomaly in (85) [61]).

其中有效普朗克质量 $M_{\rm eff}(\varepsilon)$ 依赖于总物质密度 ε。该物质密度除了包含宇宙学常数 $\Lambda = 3H^2$ 的贡献，还包含共形宇宙学的原初辐射 (但不包含因共形反常的去引力效应在 (85) 中被完全屏蔽的卡西米尔能量 [61])。

The above Euclidean-Lorentzian scenario remains valid also when the cosmological constant is effectively represented by an appropriate potential, $\Lambda \rightarrow V(\phi)/M_P^2$, of a slowly varying scalar field ϕ playing the role of inflaton [58,64]. Remarkably, this scenario automatically leads to the beginning of the inflationary evolution in the vicinity of the maximum of the potential. Thus, it resolves the main difficulty of the no-boundary prescription, according to which under the identification of Λ with the value of the potential, the most probable is the creation of the Universe at the minimum of $V(\phi)$; see Eq. (2). A vicinity of the maximum of the inflaton potential arises by a very simple mechanism. Obviously, the evolution of the inflaton ϕ in the Euclidean domain should also be periodic and subject to the Euclidean equation of motion, which can be rewritten in the form

当宇宙学常数可由慢滚标量场 ϕ (扮演暴胀子的角色 [58,64]) 的合适势 $\Lambda \rightarrow V(\phi)/M_P^2$ 有效描述时，上述欧几里得-洛伦兹暴胀场景仍然成立。值得注意的是，该场景会自动让暴胀演化始于势的极大值附近。因此，它解决了无边界假设的核心困难: 根据无边界假设，当将 Λ 等同于势的取值时，宇宙最有可能在 $V(\phi)$ 的极小处创生，参见式 (2)。暴胀子势极大值附近的区域可以通过一个非常简单的机制产生。显然，暴胀子 ϕ 在欧几里得区域的演化也应当是周期性的，并且服从欧几里得运动方程，该方程可以改写为如下形式

$$\frac{d}{d\tau}\left(a^3\dot{\phi}\right) = a^3\frac{\partial V}{\partial\phi}, \tag{98}$$

whence by integrating this equation over the instanton period of oscillations of both $\phi(\tau)$ and $a(\tau)$, one gets

由此将该方程对瞬子中 $\phi(\tau)$ 和 $a(\tau)$ 的振荡周期积分，可得

$$\oint d\tau a^3\frac{\partial V}{\partial\phi} = 0 \tag{99}$$

which means that the $\partial V/\partial\phi$ changes sign at some point inside the instanton domain, so that the instanton is always located at the extremum of the potential. We know that the evolution of both $a(\tau)$ and $\phi(\tau)$ takes place in the Euclidean time, so that $\phi(\tau)$ oscillates between two turning points in the underbarrier regime, which is possible only under the maximum of $V(\phi)$. This is a mechanism of hilltop inflation starting from the nucleation of the system from the Euclidean instanton in the vicinity of the potential maximum (Note that this mechanism resolves the old problem of hilltop inflation, when the inflaton classically rolls directly from the top of the potential and generates an infinitely large CMB amplitude. Here after nucleation from the cosmological instanton, the inflaton appears on the slope of the potential and, thus, generates a finite amplitude of the CMB spectrum [98].).

这意味着 $\partial V/\partial\phi$ 在瞬子区域内部的某点变号，因此瞬子始终位于势的极值点处。我们知道 $a(\tau)$ 和 $\phi(\tau)$ 的演化都发生在欧几里得时间中，因此 $\phi(\tau)$ 在势垒下区域的两个转折点之间振荡，这仅在 $V(\phi)$ 的极大值下方才能成立。这就是山顶暴胀的机制: 系统从欧几里得瞬子成核后始于势极大值附近 (请注意该机制解决了山顶暴胀的一个古老问题: 经典图像中暴胀子直接从势顶滚落，会产生无穷大的 CMB 振幅；而在本文中，从宇宙学瞬子成核后，暴胀子出现在势的斜坡上，因此可以产生有限振幅的 CMB 谱 [98])。

After the nucleation, the evolution consists in the fast quasi-exponential expansion during which the primordial radiation gets diluted, and the inflaton field and its energy density $V(\phi)$ slowly decay by a conventional exit scenario and go over into the quanta of conformally non-invariant fields produced from the vacuum (A realistic model should contain a sector of nonconformal fields which can be negligible on top of conformal fields in the early Universe but eventually starts dominating in the course of cosmological expansion.). They get thermalized and reheated to give a new post-inflationary radiation with a sub-Planckian energy density, $\varepsilon \to \varepsilon_{\text{rad}} \ll M_P^4/\beta$. Therefore, M_{eff} tends to M_P , and one obtains a standard general relativistic inflationary scenario for which initial conditions were prepared by the garland instanton of the above type.

成核后,宇宙演化表现为快速准指数膨胀,原初辐射在此过程中被稀释,暴胀子场及其能量密度 $V(\phi)$ 通过传统退出现象缓慢衰减，最终转化为真空产生的共形非不变场量子。(一个现实模型应当包含一个非共形场 sector，该部分在早期宇宙共形场之上可忽略，但最终会在宇宙膨胀过程中开始占据主导。) 这些量子被热化和再加热，产生具有亚普朗克能量密度的新暴胀后辐射，$\varepsilon \to \varepsilon_{\text{rad}} \ll M_P^4/\beta$ 。因此，M_{eff} 趋于 M_P ，最终得到一个标准广义相对论暴胀图景，其初始条件由上述类型的花环瞬子准备完成。

Interestingly, this model can serve as a source of quantum initial conditions for the Starobinsky R^2 -inflation and Higgs inflation theory [63,64], in which the effective H^2 is generated by the scalaron and Higgs field, respectively. In particular, the observable value of the CMB spectral tilt $n_s \simeq 0.965$ in these models can be related to the exponentially high instanton folding number, $m \simeq \exp\left(2\pi/\sqrt{3(1-n_s)}\right) \sim 10^8$, whereas the needed inflation scale in these models $H \sim 10^{-6}M_P$ determines the overall parameter $\beta \sim 10^{13}$ [63, 64] . The gigantic value of β needed to solve the problem of hierarchy between the Planck and inflation scales comprises the most serious difficulty of this scenario. The hope is that this difficulty can be circumvented by means of a hidden sector of numerous conformal fields [58,65].

有趣的是，该模型可以为斯塔罗宾斯基 R^2 暴胀和希格斯暴胀理论 [63,64] 提供量子初始条件，在这两个理论中, 有效 H^2 分别由标量子和希格斯场产生。具体而言, 这些模型中 CMB 谱倾角 $n_s \simeq 0.965$ 的可观测值可以和指数级大的瞬子折叠数 $m \simeq \exp\left(2\pi/\sqrt{3(1-n_s)}\right) \sim 10^8$ 联系起来，而这些模型中所需的暴胀尺度 $H \sim 10^{-6}M_P$ 决定了整体参数 $\beta \sim 10^{13}$ [63, 64] 。解决普朗克尺度与暴胀尺度等级问题所需的 β 的极大值是该图景最严重的困难。人们希望这个困难可以通过大量共形场构成的隐藏扇区来规避 [58,65]。

Obviously, the above formalism is capable of inclusion of such fields by replacing sums over conformal field oscillators $\sum_{\omega}$ with relevant sums $\sum_{s}\sum_{\omega_s}$ over spins s and their energies ω_s . However, a high value of β cannot be attained by a contribution of low-spin conformal fields $\beta = (1/180)\left(\mathcal{N}_0 + 11\mathcal{N}_{1/2} + 62\mathcal{N}_1\right)$, unless the numbers $\mathcal{N}_s$ of fields of spin s are tremendously high. On the contrary, this bound on β can be reached with a relatively low tower of higher-spin fields, because a partial contribution of spin s to β grows as s^6 [135]. The solution of the hierarchy problem thus becomes a playground of the $1/\mathcal{N}$ -expansion theory for a large number $\mathcal{N}$ of conformal species. However, an additional problem arises associated with the fact that such theories acquire reduced gravitational cut-off $\Lambda_{\text{grav}} \sim M_P/\sqrt{\mathcal{N}}$ [136-139], above which an effective field theory stops working, and the needed value of inflation scale might exceed this cut-off. Fortunately, due to a peculiar property that the number of polarizations of higher-spin conformal particles $\mathcal{N} \sim s^2$ grows with spin much slower than $\beta \sim s^6$, this difficulty is possible to circumvent. If the hidden sector is built of higher-spin conformal fields (CHS) [65], then the known gravitational cut-off Λ_{grav} turns out to be several orders of magnitude higher than the inflation scale. This justifies the omission of the graviton loop contribution and the use of the above nonperturbative (conformal anomaly-based) method.

显然，上述形式体系可以容纳这类场，只需将共形场振子 $\sum_{\omega}$ 的求和替换为针对自旋 s 及其能量 ω_s 的对应求和 $\sum_{s}\sum_{\omega_s}$。不过，低自旋共形场 $\beta = (1/180)\left(\mathcal{N}_0 + 11\mathcal{N}_{1/2} + 62\mathcal{N}_1\right)$ 无法获得大的 β 值，除非自旋 s 场的数量 $\mathcal{N}_s$ 极高。相反，若存在相对低阶的高自旋场塔，则可达到 β 的这个限值，因为自旋 s 对 β 的部分贡献随 s^6 增长 [135]。因此，层级问题的求解成为适用于大量共形种类 $\mathcal{N}$ 的 $1/\mathcal{N}$ 展开理论的研究领域。但随之出现一个额外问题: 这类理论的引力截断 $\Lambda_{\text{grav}} \sim M_P/\sqrt{\mathcal{N}}$ 会降低 [136-139]，有效场论在截断以上不再成立，而我们所需的暴胀尺度可能超过该截断。幸运的是，由于高自旋共形粒子 $\mathcal{N} \sim s^2$ 的极化数随自旋增长的速度远慢于 $\beta \sim s^6$ 这一特殊性质，这个困难可以规避。如果隐秘扇区由高自旋共形场 (CHS) 构成 [65]，那么已知的引力截断 Λ_{grav} 会比暴胀尺度高出数个数量级。这就证明忽略引力子圈贡献、使用上述非微扰 (基于共形反常的) 方法是合理的。

The further development of this concept has shown that this model of initial conditions suggests many interesting physical predictions. They include a potentially observable thermal imprint on the primordial CMB spectrum [61,134], a new type of the hilltop inflation [63, 64] arising in the synthesis of the Higgs inflationary model with a large nonminimal inflaton coupling considered above, etc. Quite interestingly, for the above picture of hierarchy between the Planck and the inflation scales, the thermal correction to the spectral parameter of CMB $\Delta n_s^{\text{thermal}} \sim -0.001$, depending on the properties of the so-called hill-like inflaton potential, might appear in the third decimal order [63, 134] - the precision anticipated to be reachable in the next generation of CMB observations following Planck. This means that a potential resolution of the hierarchy problem in the CFT scenario via CHS simultaneously would make measurable the thermal contribution to the CMB red tilt. This contribution will be complementary to the most fundamental observational evidence for inflation theory - red tilt of the primordial CMB spectrum caused by the deviation of the slow roll evolution from the exact de Sitter scenario [97, 98].

这一概念的后续发展表明，这个初始条件模型给出了许多有趣的物理预言。其中包括原初 CMB 谱上可能被观测到的热印记 [61,134]，以及前文讨论的、具有大非最小暴胀子耦合的希格斯暴胀模型结合后产生的新型山顶暴胀 [63, 64] 等等。非常值得注意的是，对于上文普朗克尺度与暴胀尺度的层级图景，CMB 谱参数 $\Delta n_s^{\text{thermal}} \sim -0.001$ 的热修正取决于所谓山顶型暴胀势的性质，其大小可达千分位量级 [63, 134] ——这正是普朗克之后下一代 CMB 观测有望达到的精度。这意味着，在 CFT 框架下通过 CHS 解决层级问题的同时，CMB 红倾斜的热贡献也将可以被测量。该贡献会补充暴胀理论最基本的观测证据——即由慢滚演化偏离精确德西特场景导致的原初 CMB 谱红倾斜 [97, 98]。

Finally, there is one more important comment on this model of initial conditions. Note that this scenario is possible only for spatially closed cosmologies, because it is the presence of the positive curvature term $k/a^2, k = +1$, that allows the existence of two turning points for the solution of effective Euclidean equations of motion. For spatially flat or open model with $k = 0$ or -1, both turning points do not exist for positive real a^2 (note that Eq. (88) then goes over into $a_\pm^2 = \left(k \pm \sqrt{1-4CH^2}\right)/2H^2 < 0$), so that there is no transition from the periodic motion in the classically forbidden domain to the phase of cosmological expansion with the Lorentzian spacetime signature. Phenomenologically, this sounds disturbing because inflation is usually assumed to be considered in a spatially flat Universe, and its flatness is considered as one of the advantages of the inflation scenario, matching very well with observations. Thus, the idea of non-flat inflationary cosmology is not popular, even though relevant models of inflation have been worked out [140- 143]. However, as it is recently observed in the exhaustive treatment of the Planck 2018 CMB temperature and polarization data [144, 145], these datasets are now preferring a positive curvature at more than the 99% confidence level with a mean $\Omega_K \simeq -0.04$. Even though this preference of a closed Universe is associated with discordances arising for local cosmological observables, known as Hubble tension [146], robust observational evidence in favour of a positive spatial curvature serves a strong motivation for the suggested model of quantum initial conditions.

最后，针对这一初始条件模型，还有一个重要补充说明。注意该场景仅适用于空间闭合宇宙学，这是因为只有存在正曲率项 $k/a^2, k=+1$ ，有效欧几里得运动方程的解才会存在两个拐点。对于空间平直或开放模型，对应曲率为 $k=0$ 或-1，当 a^2 为正实数时不存在任何拐点 (此时式 (88) 可转化为 $a_\pm^2 = \left(k \pm \sqrt{1-4CH^2}\right)/2H^2 < 0$)，因此经典禁戒域内的周期运动不会转变为洛伦兹时空符号下的宇宙膨胀阶段。从唯象学角度来看这一结论并不理想，因为通常 inflation 假设发生在空间平直宇宙中，宇宙平直性是 inflation 场景的核心优势之一，和观测结果吻合极好。因此非平直 inflation 宇宙学的相关研究并不热门，即便已有学者构建出相应 inflation 模型 [140-143]。然而，近期对普朗克 2018 年 CMB 温度与极化数据的全面分析显示 [144, 145]，上述数据集在 99% 以上置信水平更支持正空间曲率，平均值为 $\Omega_K \simeq -0.04$ 。尽管闭合宇宙的这一偏好和局域宇宙学观测结果存在不一致，即所谓的哈勃张力问题 [146]，但支持正空间曲率的可靠观测证据，仍然为本文提出的量子初始条件模型提供了强有力的依据。

Concluding Remarks

结语

We have presented various aspects and applications of perturbative quantum gravity in the physics of the early Universe and quantum cosmology. Starting from somewhat naive normalization requirements for the cosmological wave function, establishing certain restrictions on particle phenomenology, we have amounted to the links between the physics of a cosmic microwave background at hundreds of megaparsecs scale and the physics of an electroweak sector of the Standard Model of particles and fields at the 10^{-16} cm scale. The established relation between the parameters of the CMB spectra and Higgs boson mass clearly violates a well-known physics principle of separation of scales and exists entirely due to such a perfect microscope existing in nature as cosmological expansion - apparently the most significant phenomenon in the Universe.

我们介绍了微扰量子引力在早期宇宙物理学和量子宇宙学中的多个研究方向与应用。从对宇宙波函数的基本归一化要求出发，我们得到了粒子唯象学的若干限制，最终建立了百兆秒差距尺度宇宙微波背景物理学与 10^{-16} cm 尺度粒子与场标准模型电弱 sector 物理学之间的关联。所得到的 CMB 谱参数与希格斯玻色子质量之间的关联明显违背了著名的标度分离物理学原理，而这一关联完全源自宇宙膨胀——这个自然界中最完美的显微镜，它显然是宇宙中最重要的现象。

Moving further down to the past in the history of the Universe, we put forward a conjecture of its micro-canonical density matrix as a quantum state which describes the quantum origin of the inflationary stage of cosmological evolution. Its construction, conceptually stemming from the Occam razor principle of avoiding redundant assumptions, is based on the path integral representation of a generic solution of Wheeler-DeWitt equations and turns out to be very productive in application to the Universe dominated by conformally coupled matter fields. The density matrix paradigm allows one to solve such a problem as elimination of infrared catastrophe in the theory of the no-boundary quantum state of the Universe and suggests initial conditions for inflation in the form of a special quasi-thermal cosmological instanton.

进一步回溯宇宙的演化历史，我们提出了宇宙微正则密度矩阵猜想，将其作为描述宇宙暴胀阶段量子起源的量子态。该构造从概念上源自避免冗余假设的奥卡姆剃刀原理，建立在惠勒-德维特方程通解的路径积分表示基础上，当应用于共形耦合物质场主导的宇宙时成效显著。密度矩阵范式可以解决宇宙无边界量子态理论中的红外灾难消除问题，并以特殊准热宇宙瞬子的形式给出了暴胀的初始条件。

Conceptually, the microcanonical density matrix of the Universe suggests the reincarnation of the Big Bang scenario on a qualitatively new level of its understanding. The inflation paradigm replaced the notion of an infinitely hot and dense initial state of the Universe by an effectively vacuum state at zero temperature, whose quantum fluctuations eventually gave rise to a large-scale structure of the Universe. In the density matrix paradigm, we again get back to the notion of an initially hot Universe originating from the classically forbidden state of the gravitational field, but characterized by finite rationally manageable parameters of effective temperature, size and rate of expansion.

从概念上看，宇宙的微正则密度矩阵让大爆炸理论在全新的理解层面获得了重生。暴胀范式已经将最初无限热、无限致密的宇宙初始态替换为零温下的有效真空态，该态的量子涨落最终形成了宇宙的大尺度结构。在密度矩阵范式中，我们重新回到初始热宇宙的概念——它起源于引力场的经典禁戒态，但由有限、可合理处理的有效温度、尺度和膨胀率参数描述。

The quantum origin of the Universe might be a sub-Planckian, even relatively low-energy, phenomenon, whose description then may be possible within our semiclassical perturbative methods. The success of this conjecture depends on the construction of a hypothetical particle physics model of numerous higher-spin conformal fields, which would form a theoretical framework for the solution of a quantum initial condition problem in cosmology. Moreover, as it stands it also might serve as a selection criterion for the landscape of stringy vacua. The string theory, as is well known, incorporates an infinite tower of higher-spin fields which, however, are massive and cannot be conformally coupled to spacetime metric. On the other hand, there is a popular idea that strings might be a broken phase of the higher-spin theory [147, 148]. This might open prospects of unification of the suggested ideas with the fundamental origin of particle physics phenomenology necessary for the solution of the Planck vs inflation scale hierarchy problem.

宇宙的量子起源可能是一个亚普朗克的、甚至能量相对较低的现象，因此我们可以用现有的半经典微扰方法对其进行描述。这一猜想的成立取决于能否构建一个包含大量高自旋共形场的假想粒子物理模型，该模型将为解决宇宙学量子初始条件问题提供理论框架。此外，就目前而言，它还可以作为弦真空景观的选择标准。众所周知，弦理论包含无穷多的高自旋场 tower，但这些场都是有质量的，无法与时空度规共形耦合。另一方面，有一个广为接受的观点认为弦可能是高自旋理论的破缺相 [147, 148]。这或许为将本文提出的观点与粒子物理唯象学的基本起源结合起来打开了前景，而这正是解决普朗克尺度与暴胀尺度等级问题所必需的。

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